Price to Earnings (P/E) Determinants and the Valuation of Private Firms: A Cross-country Comparison

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Declaration

Whilst registered as a candidate for the above degree, I have not been registered for any other research award. The results and conclusions embodied in this thesis are the work of the named candidate and have not been submitted for any other academic award

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Acknowledgements

Although this thesis is a product of my own work, I felt the need while writing it, to use the word “we” instead of “I” for two reasons. The first one, and perhaps the most important, is that I felt that I could not have done this without the help and support of many people, which I will thank individually below. The second one, is because I wanted by using the first-person plural, to make the text as inviting, to any potential reader, as possible.

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Abstract

Private firms are the predominant form of incorporation in both the developed and the developing world. They were, however until recently under-researched, mainly due to the lack of data availability. To overcome this difficulty, both researchers and academics approximate private entities through public equivalents. One of the inputs that is needed to do that is the discount rate, by which the cash flows, from investing in a private firm, need to be adjusted to account for the risk that such investments entail.

The main focus of this thesis is to carefully examine the variables that have been identified throughout the literature, as being impactful to the discount rate. These factors are examined through the scope of a novel methodology and will allow appraisers to have a framework of reference when valuing a private company. The purpose is not only to highlight the determinants of the Price to Earnings (P/E) ratio in the private companies' valuation however, but to do so in an international setting, as both the UK and the US are examined, in an attempt to document differences in the risk profiles of investors from these countries.

To determine the level at which each of the factors, affects the discount rate, Principal Component Analysis is employed, on public companies, selected to proxy private firms. This methodology is used to reduce the size of extensive datasets, while retaining most of their variability. The components produced, are linear combinations of the original variables, devoid of any multicollinearity issues inherent in large datasets. These components are then regressed against a valuation proxy, a Price-to-Earnings ratio, calculated initially for the public companies' dataset and later, for a private companies' set, the latter being adjusted to account for the illiquidity discounts exclusive to private enterprises during the Mergers and Acquisitions process.

The results indicate that a private company's discount rate can be approximated best with the inclusion of the Free Cash Flows (FCF), the Debt-to-Equity, Assets, Cost of Debt, Total Beta, Z-Score, Auditors and Jensen's Alpha for the UK and Earnings Before Interests Taxes Depreciation Amortization (EBITDA), external shocks (Financial Crisis), FCF, Debt-to-Equity, Return on Capital, Percentage of Insiders Holding Stock, Beta, Tax Rate, Marginal Profit (MPK) and Jensen's Alpha for the US. Investors from the UK, appear to be more risk-averse than their US counterparts, as they seem to value traditional variables more, than the ones focused on profitability, earnings and debt.

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List of Abbreviations

Abbreviations	Meaning
AIM	Alternative Investment Market (LSE)
APT	Arbitrage Pricing Theory
APV	Adjusted Present Value
BEA	Bureau of Economic Analysis
BVD	Bureau Van Dijk
Cap	Capitalization
CAPM	Capital Asset Pricing Model
CCAPM	Consumption Capital Asset Pricing Model
CCI	Consumer Confidence Index
CEO	Chief Executive Officer
CSR	Corporate Social Responsibility
DCF	Discounted Cash Flow
EAFE	Europe Australia and Far East
EBITDA	Earnings Before Interest, Taxes, Depreciation and Amortization
EMRP	Equity Market Risk Premium
EPS	Earnings per share
ERP	Equity Risk Premium
EU	European Union
EV	Enterprise Value
FAS	Financial Accounting Standards
FCFE	Free-Cash-Flow-to-Equity
FCFF	Free-Cash-Flow-to-Firm
FDI	Foreign Direct Investment
FE	Fixed-Effect
FTSE	Financial Times Stock Exchange Index
GBP	Pound Sterling
GDP	Gross Domestic Product
GGM	Gordon Growth Model
HML	High minus Low
IAS	International Accounting Standards
ICAPM	International Capital Asset Pricing Model
ICB	Industry Classification Benchmark
ICRG	International Country Risk Guide
IFRS	International Financial Reporting Standards
IK	Investment to Capital
IPCPL	Implied Private Company Pricing Line
IPO	Initial Public Offering
IRP	Industry Risk Premium

JOBS Act	Jumpstart Our Businesses Act
LRA	Linear Regression Analysis
LSE	London Stock Exchange
M&A	Mergers and Acquisitions
MID	Minority Interest Discount
MLE	Maximum Likelihood Estimation
MLR	Multiple Linear Regression
MPK	Marginal Profit
MSCI	Morgan Stanley Capital International
NACVA	National Association of Certified Valuators and Analysts
NASDAQ	National Association of Securities Dealers Automated Quotations
NBER	National Bureau of Economic Research
NOMAD	Nominated Advisor
NYSE	New York Stock Exchange
OECD	Organization for Economic Co-operation and Development
OLS	Ordinary Least Square
ONS	Office for National Statistics
OTCBB	Over-The-Counter Bulletin Board
P/B	Price to Book
P/E	Price to Earnings
P/S	Price-to-Sales
PCA	Principal Component Analysis
PLS	Partial Least Square
PV	Present Value
R&D	Research and Development
REIT	Real Estate Investment Trust
ROA	Return on Assets
ROC	Return on Capital
ROE	Return on Equity
S&P	Standard and Poor’s
SCR	Company Specific Risk
SEC	Securities and Exchange Commission
SFAS	Statement of Financial Accounting Standards
SIC	Standard Industrial Classification
SME	Small and Medium-sized Enterprises
SOX	Sarbanes-Oxley
SP	Size Premium
SSR	Sum Square of Residuals
TCoE	Total Cost of Equity
TVM	Time Value of Money
UK	United Kingdom
US (A)	United States of America
US GAAP	United States' Generally Accepted Accounting Principles

VAR	Vector Autoregression
VIF	Variance Inflator Factors
WACC	Weighted Average Cost of Capital

1. Introduction

1.1 Overview

Private companies are an important driver of the global economy. Several recent studies (Asker, Farre-Mensa, and Ljungqvist, 2015; Cooper and Priestley, 2016; Gilje and Taillard, 2016), attest to that idea, by pointing towards the fact that in the US, private firms accounted for almost 95% of all businesses in 2008, with their net income approaching 54% of the total net income from all enterprises . The variety in the size of private firms is also stunning. Abudy, Benninga, and Shust (2016) report that although many private firms are small, 86.4% of all firms that employed more than 500 people were unlisted, and with the wages paid to their employees represent almost half of the country's GDP. Similar is the situation in the UK, with private businesses accounting for more than 97.5% of all businesses, while holding 67% of the total corporate assets in the country (Brav, 2009) and with unlisted companies accounting for more than 50% of the GDP (Michaely and Roberts, 2012). These facts however raise the question of how, to date, such an important part of the economy could be so neglected by researchers in the finance arena.

To answer this question, one has to turn to the related literature and to studies like those of Hope, Thomas, and Vyas (2013) and Brav (2009), who attribute this shortcoming to the lack of readily available data on private businesses. Other issues, like the unique structure of ownership, or the illiquidity faced by private firms, further amplified the problem. All of these issues are also reflected in the valuation of private firms and in the discounts imposed on transactions that involve them. Specifically, faced with apparent uncertainty, investors tend to require higher discounts. Officer (2007) reports discounts on private companies' M&A transactions that can represent up to one third of the final value.

The information asymmetry between investors and managers in unlisted firms is further accentuated by the fact that over the past decades, the discount rate literature has identified a great number of factors that are prone to affect expected returns. As we will go into detail on this topic in the section that follows (and in increased depth in the extensive literature review of this thesis), we will just briefly review the evolution of the expected returns' bibliography, starting with the original version of the CAPM (and the several sub-

categories of it that sprung into development over the years like the CCAPM and the ICAPM), and going up to Ross's Arbitrage Pricing Theory (Ross, 1976) and Fama - French multifactor models (Fama and French, 1995, 2012).

All of the above compose a complicated picture for the field of the discount rates, especially in conjunction with the valuation of private enterprises. A situation which we endeavor to untangle with this thesis and hopefully add further critical insight on these issues, which will serve as a point of reference for future research.

1.2 Theoretical background and research context

As we touched briefly on several issues in the previous section, we feel that it is important to provide a brief overview on how the literature has evolved and how it affects the topics that will be discussed throughout this thesis. The first idea we want to present is the way that we view the valuation process. Value can be estimated through different methodologies, each with its distinct characteristics. Specifically, we have the asset-based methodology, the comparable approach and finally the Discounted Cash Flow method. According to several sources the DCF methodology is the most appropriate one, as not only does it produce the most efficient and effective results from all three methods (Kaplan and Ruback, 1995), but also because it allows for several different inputs to be incorporated in the final estimate (Damodaran, 2012). This is the reason why even when other methodologies are employed, a DCF estimation is usually performed at some stage in the valuation process.

The inputs we referred to previously are the cash flows that an investor can expect from the investment, its growth rate and most importantly the discount rate, which will be the centerpiece of this thesis, in an attempt to better outline the factors that determine it. National Association of Certified Valuators and Analysts (2012) guidelines suggest that an analyst should consider the overall market environment the business operates in, the industry and the competition and finally the company's own performance as a means to

determine the discount rate. The focus on the determinants of the discount rate, characterized here through the lens of the P/E ratio, in itself points in itself points towards the shift in the literature mentioned in Cochrane (2011), from the expected cash flows towards the discount rate and what it represents.

The academic literature has evolved over the years following exactly these ideas. As the distinction between systematic and unsystematic risk became more evident, it resulted in the idea of the CAPM (based on the work of Lintner, 1965b; Mossin, 1966; Sharpe, 1964; Treynor, 1961) and gave birth to the Beta. It was soon evident however that the Beta was not enough to interpret the expected returns of investing in a firm. Thus other models were developed as improvements to the CAPM (see for instance the Beta adjustment proposed by Blume (1971), the leverage one proposed by Hamada (1972), the ICAPM (Fernandez, 2005), etc.). A particular strand of models was based originally on the work of Ross (1976) and his Arbitrage Pricing Model and later on that of Fama and French, who developed the original multifactor model (Fama and French, 1993), that incorporated the size as well as the market value as factors that need to be considered in developing the discount rate.

The Fama-French model gave a significant push to the literature towards this direction. Since its original inception several additional factors have begun to emerge. We have determinants that are akin to macroeconomic indicators, with GDP growth, unemployment rate, inflation, interest rates and momentum being the most prominent amongst them (Chordia and Shivakumar, 2006; Durré and Giot, 2007; Hasan, 2008; Lemmon and Portniaguina, 2006; Rangvid, 2006). Others are related to the fluctuations within the industry that a company is part of and include topics that range from competition and barriers to entry, to legislation (Chemmanur and He, 2011; Klapper, Laeven, and Rajan, 2006). Finally, we have those that are specific to the companies themselves (or the unsystematic portion of the risk), and include variables that range from management and governance related (Chen, Harford, and Li, 2007; Gao and Zhang, 2015; Leone, Wu, and Zimmerman, 2006) to others, which are more focused on the performance of the firm (Anderson and Brooks, 2006; Campello, 2006)

One of the most important recent developments, however, in the cost of capital theory, as far as unlisted companies are concerned, is the Total Beta, an idea originally developed by Damodaran (1999a), that allows analysts to estimate the risk associated with a private company (an idea which we will extensively cover in the Literature Review section of the thesis). Based upon that, another theoretical model, has been developed by Dohmeyer, Butler, and Burkert (2014), the Implied Private Company Pricing Line model. The underlying idea is, that there are no arbitrage opportunities between private and public equity market. Thus, the value of privately held companies is a function of the risk-adjusted discount rate of public companies, further adjusted by the private companies' transaction costs and the regulation, or lack thereof, in the private equity market.

1.3 Motivation and Contribution to the Literature

The motivation for this attempt to expand knowledge in the field of the discount rates, was the constant evolution of new ideas over the past few years, as explained previously, particularly for the private companies' aspect of it. Specifically, private companies research has seen a considerable growth, not only due to the fact that data on them are becoming easier to acquire, but also because the restrictions that were related to their structure, both in terms of ownership and capital structure, are now viewed as opportunities to test the validity of popular theories, instead of constraints. This is something that will be further explained in the private companies' sample on the analysis section, however we want to provide an early glimpse on how this change has occurred.

The first study we will mention here, which has also served as a motivational pillar for this work, is the seminal study of Brav (2009), which is perhaps one of the most cited papers in the relevant literature. The author explores what the optimal capital structure for private enterprises is and finds differences between private and public firms. Specifically, he suggests that debt is the main source of financing for private firms. He also attests to the notion that equity financing is costlier, mainly due to information asymmetry concerns,

as well as, due to their managers (who often are the owner of the firms) unwillingness to share control of their firms. The earlier study of Officer (2007), pointed out however that debt capital in private firms leads to higher acquisition discounts compared with public firms (approximately somewhere between 15% to 30%), when these companies are engaged in M&As. What is clear from studies such as these, is that although public firms are often used as paradigms for the valuation of equivalent (in terms of size and industry) private ones, the process can be treacherous and lead to less than accurate results.

Besides its focus on private companies, the study of Brav had another positive "side-effect". It inspired several studies, including this one, to shift their attention on private companies from the UK, and not look exclusively into the US market. Michaely and Roberts (2012) and later Allen, Carletti, and Marquez, (2014), argue that the UK companies' samples have the advantage to be more populated than their US counterparts. Furthermore, as they explain the UK and the US share many similarities in terms of the overall economic environment. This was also something we considered regarding the implications of this research, as we wanted to examine whether the results on the factors that affect the UK discount rates would be similar to those of the US. Discrepancies would be indicative of different risk profiles for investors from these two countries.

Another study that has been pivotal for this research is the survey paper, of Cochrane (2011). This study is focused primarily on summarizing the developments in the discount rate literature, beginning from the dividend yield regressions, and how those were interpreted by the expected returns of stocks, going through the Beta models that were developed more than fifty years ago, with the CAPM being the most focal point of research in the beginning and later the multifactor models, that gave birth to a great number of factors in the asset pricing literature. The main reason, however, that Cochrane's study was important for this one is for two major points that are made. The first one is that the discount rates can serve as the best proxy to asset pricing, regardless of the asset class (whether we are talking about stocks and bonds or whole companies' value is irrelevant in this context), as they can best describe the over-time variation in those assets' value.

The second, and of utmost importance to this study, as it is related to the methodological approach, is that discount rates (or expected returns) are tied to asset pricing through the various assets characteristics' covariance. As we will see in the methodological portion of the thesis the methodology takes advantage of the covariance between the various elements that have been used to describe the discount rates, to enable an articulation and characterization of the most important ones. A similar idea to that of Cochrane has been mentioned in an earlier survey paper of Constantinides (2002), who also suggests that the covariance should be the focal point of future asset pricing research.

After this brief explanation on how this thesis is affected by the relevant literature the motivation has been crystallized. What we want to achieve is to primarily consolidate all the factors that have been given prominence throughout the literature and determine how each of them contributes to the formulation of the discount rate in the valuation of private companies. The goal is to build a theoretical framework that will approximate a private company's value as efficiently and effectively as possible, while simultaneously shedding some light on how the link between private and public companies can be best utilized (which public companies can best serve the role of a proxy in the valuation process). The choice of public comparable firms, is firmly rooted within the literature, as we follow studies such as the one by Gerakos, Lang, and Maffett (2013), who focus on AIM and NASDAQ companies (among others), as those best represent the constraints faced by unlisted firms.

Our work is closely related to a recent strand in the literature namely to the studies of Abudy et al. (2016),Asker et al. (2015), Brav (2009), Cooper and Priestley (2016), Hope et al. (2013), Michaely and Roberts (2012) and Sheen (2016). The results will have an impact on both academics and practitioners alike. For academics it will extend the relevant literature on the discount rates and the cost of capital for private businesses, which is a topic that has only gained traction in recent years, as data become more readily available. Finally, we hope that it will also provide a foundation for analysts which can be used in their day to day activities.

1.4 Statement of objectives and Research Methods

Based on the previous discussion, it has been established that the valuation of private companies is an area that has been the recipient of less attention than their public counterpart, despite the fact that private companies are of equal, if not higher importance to public ones. As it is mentioned in several studies however, the process of valuation is not ruled by a specific set of factors that need to be taken into consideration (Asker et al., 2015; Gilje and Taillard, 2016; Sheen, 2016). This leads analysts to adopt different approaches, which in turn results in dissonance on how the discount rates impact on the value of those firms.

To illustrate that point, one can consider that, the National Association of Certified Valuators and Analysts (2012) indicates that performing a valuation of a closely held business is not a trivial matter, as there is no established consensus methodology or agreed procedure on how it needs to be done, and methods vary by both analysts' and firms' differential approaches. What further contributes to this issue is also what Cochrane (2011) refers to as a 'zoo of factors' for the determination of the discount rate, which according to him is what research should be focused on, as the discount rates better describe the price of any asset.

Therefore, this research will set out to:

1. Review the relative literature in order to evaluate the methodologies employed to perform valuation, in both public and private markets, in the major financial markets of the USA and UK.
2. Evaluate, in this contextual framework, the relationship between risk and return, and how it is established in the valuation process through the discount rates applied.
3. Distil, catalog and sort all the risk factors incorporated in the discount rate, as they are highlighted in the related literature. Incorporating a full and systematic evaluation and analysis of unsystematic risk and its constituent elements.
4. Highlight any differences in the way investors in the UK and US perceive the risks associated with investing in private enterprises.

5. Use quantitative testing techniques to construct a framework under which business appraisers will be able to assign an appropriately informed and scientifically defensible discount rate for private companies.

In order to address all of these ideas, we will continue the thesis by reviewing the literature, isolating the factors that contribute the most to the variation of the discount rate (as those will be highlighted through the PCA methodology), and finally examine the effects these factors have on a unique sample of, initially, public, and secondarily, private companies. All of the above will be approached through the structure described, in detail, in the section that follows.

1.5 Thesis Structure

This thesis is structured as follows (as shown in Figure 1 that follows). In this first chapter we provided a preliminary overview on how some key ideas that were developed in the literature, spurred an interest in the field, and provided not only the motivation, but also the theoretical framework, on which this thesis will be based upon. We felt that it was important to highlight a specific set of studies, in both the fields of discount rate and private companies, as those act as the link between these two topics and will serve, in this way, as the basis for the development of the ideas contained herein.

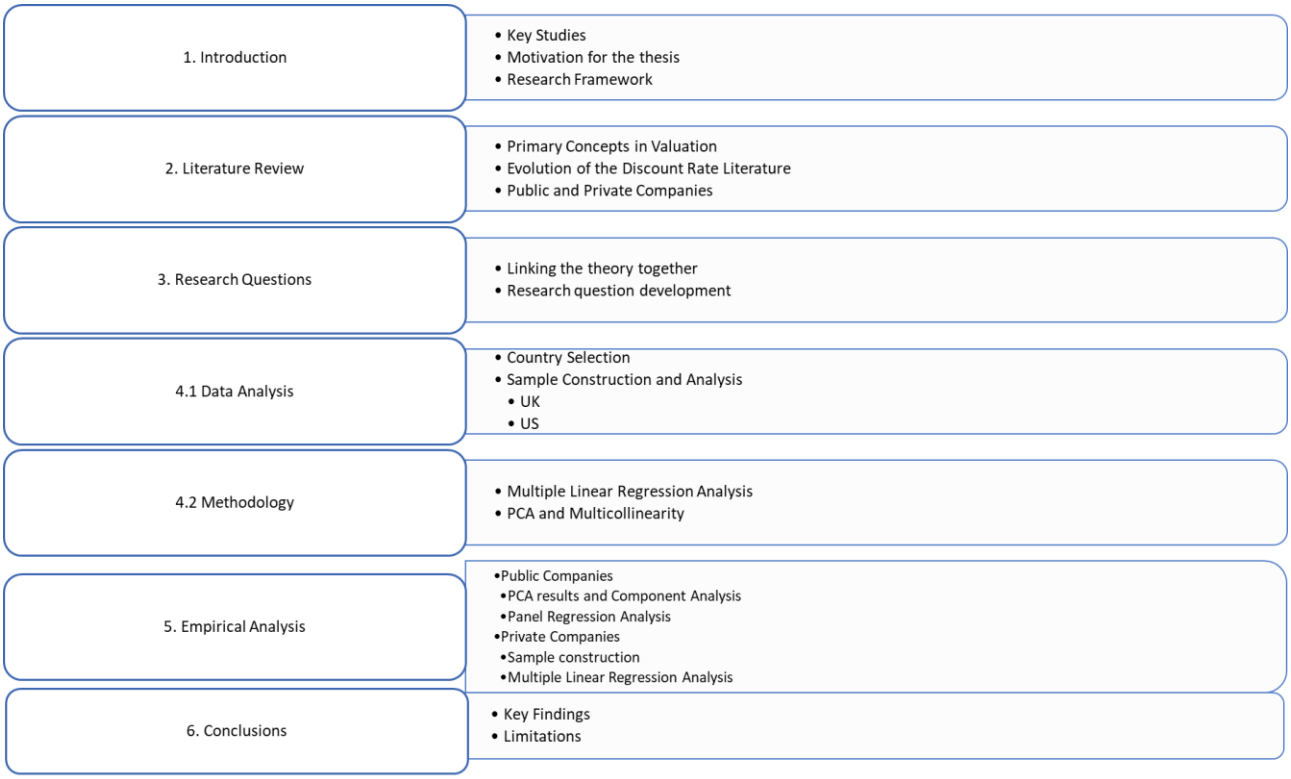


Figure 1: Overview of the thesis' structure

In the second chapter, we will go over the literature in as much detail as possible. We will begin the review, by providing a detailed description on what constitutes value, what methods of valuation have been used in both a public and private setting, and ultimately why the valuation process is such an important concept in the Accounting and Finance literature. We will then cover other important topics, starting with the concept of risk and return and how risk can be analyzed to its systematic and unsystematic components. We will provide a detailed analysis on both those topics and look into the ideas that have been developed around them, covering notions that span from the CAPM and the rise of the Betas, to the other methodological frameworks that were developed to counter the Betas' shortcomings, namely theoretical models such as the Arbitrage Pricing Theory (APT) and the Multifactor Models. We will then critique studies that refer to the unsystematic portion of risk and provide guidance and direction through the plethora of ideas that have been developed over time. We will finally provide a detailed record on how public and private companies are linked and present the latest developments in the cost of capital research for private enterprises.

This overview of the literature will then allow us to transition to the next part of the thesis, the research questions, that will be covered in the third chapter. In this section we will begin by summarizing the basic concepts and ideas that were developed in the literature review, and how those raise the research questions, which we will address through the empirical analysis. This part of the thesis is highly important, not only because it provides the reasoning behind the motivation for the analysis, but also because it ties in the ideas developed by previous research. As the number of factors that have to be taken into consideration is significant, it is only natural for the research questions to encapsulate this complexity and lead on to the next part of the thesis.

Having built the foundations on the literature, we turn towards constructing the sample and explaining the methodological approach in the fourth chapter. In this part, we will explain the reasoning behind the decision to conduct a comparative study, the countries we selected to compare, and how we used the variables defined in previous papers, in order to construct the sample. We consider in depth how the three most common areas of valuation were approached (macroeconomic, industry and company-specific factors) and how these are constructed in the thesis. We will also provide a detailed account for the sample and how its structure relates to relevant studies. In the second half of this chapter, we will deliver an overview on the most common methodological approaches in the discount rate field, explain common issues that might arise and how the methodology chosen (PCA) counters them. We will also do an in-depth analysis of the methodology and consider the mathematical concepts that define it.

The fifth section of the thesis is dedicated to the empirical analysis. We will begin by performing a simple linear regression analysis in order to exemplify the inadequacy of the methodology to provide us with inference on how the valuation process is affected by the multitude of variables defined in the previous section. This problem will be highlighted further by performing a variance inflator factor analysis to determine the level of multicollinearity issues present in studies with a large number of determinants. We will then use the methodology explained in chapter 4, namely PCA, to construct the new set of variables that are devoid of this issue. We will at this point analyze and explain how the products of this methodology,

which are called components, are being formed and what each one of them represents. We will then use these components as the new independent variables in panel regression analysis. This will allow us to determine which component variables contribute the most in the valuation process of the public firms (which are comparable to private ones).

The final part of the analysis will be dedicated to the private businesses. We will provide the relative information on how the dependent variable (the valuation ratio) was determined, and how the discount rate was measured, based on a series of research papers. We will also explain how we utilized the components that were highlighted by the analysis we conducted on the public companies (of the AIM and NASDAQ), for private companies that were going through the M&A process. It is important to understand the limitations we faced when constructing the private companies' sample and how we matched it with the public companies' one, and also have an idea of how we addressed the basic problems that are associated with private entities' valuation, mainly through the addition of variables or through transformations in the discount from the M&A deals. We will finally perform a multiple regression analysis to verify which of those variables, identified through the methodology, are important to be used in the valuation process of private businesses.

At this point the intensity of the data captured in this thesis, will be evident. At the first stage the data will range from macroeconomic, drawn from sources such as, but not limited to, the OECD, the Bank of England, CIA's website on geopolitical characteristics, to industry and company specific data, sourced from Bloomberg and Capital IQ among others. These will include key economic indicators (for instance we will be including GDP growth, interest rates, inflation, etc.) industry adjusted variables, as well as, several measures of company performance, ranging from management and governance (e.g. how stock is distributed amongst the insiders) to financial statement-related elements (Assets, EBITDA, FCFE and FCFF, etc.).

As we have explained, however, with this thesis we set out to determine how the discount rate for private companies can be approximated. For that reason, we will need to include, besides our primary dataset, two

more sources of data. Based on prior literature, we will be matching our public companies' sample, based on a set of characteristics that will be explained in greater detail in the relevant sections of the thesis, with two different datasets. The first one will be, as expected, a private companies' sample, which will be drawn through several sources but primarily from Bureau Van Dijk's (BVD) database that will be matched with the components from the PCA, and will be composed of private companies that have undergone an M&A. This will allow us to form our independent variables for the MLR analysis that we will perform at the relevant empirical stage. For our dependent variable however, we will utilize a separate dataset from Bloomberg and Capital IQ, which will be comprised of public M&As, which will also be matched to our private companies' M&As. This is done so that we can compute the difference between public and private M&A deals, and subsequently the illiquidity discount, that private companies' transactions are impacted by.

The last part of the thesis is where we draw final conclusions based on the previous analysis and how the research questions have been addressed. We will also provide an idea of the limitations we faced through the study, and what can be done based on the thesis to further expand knowledge in the discount rate field.

2. Literature Review

The literature review focuses on the critical factors which have been found to impact rates of return with a view to addressing the intractable area of assessing discount rates for private company investments. Before embarking on the journey of evaluating the literature, it is imperative that we create a road map on the issues that will be addressed throughout the review, and how the ideas that will be presented are linked to the focus of this thesis, which is the valuation of private companies. To assist with this task we created a Papineau's Literature Tree (the reasoning behind the tree is explained by Ryan, Scapens, and Theobald, 2002), as exhibited in the Figure 2 below.

From this graphical representation it becomes evident that the literature on the derivation of value is very

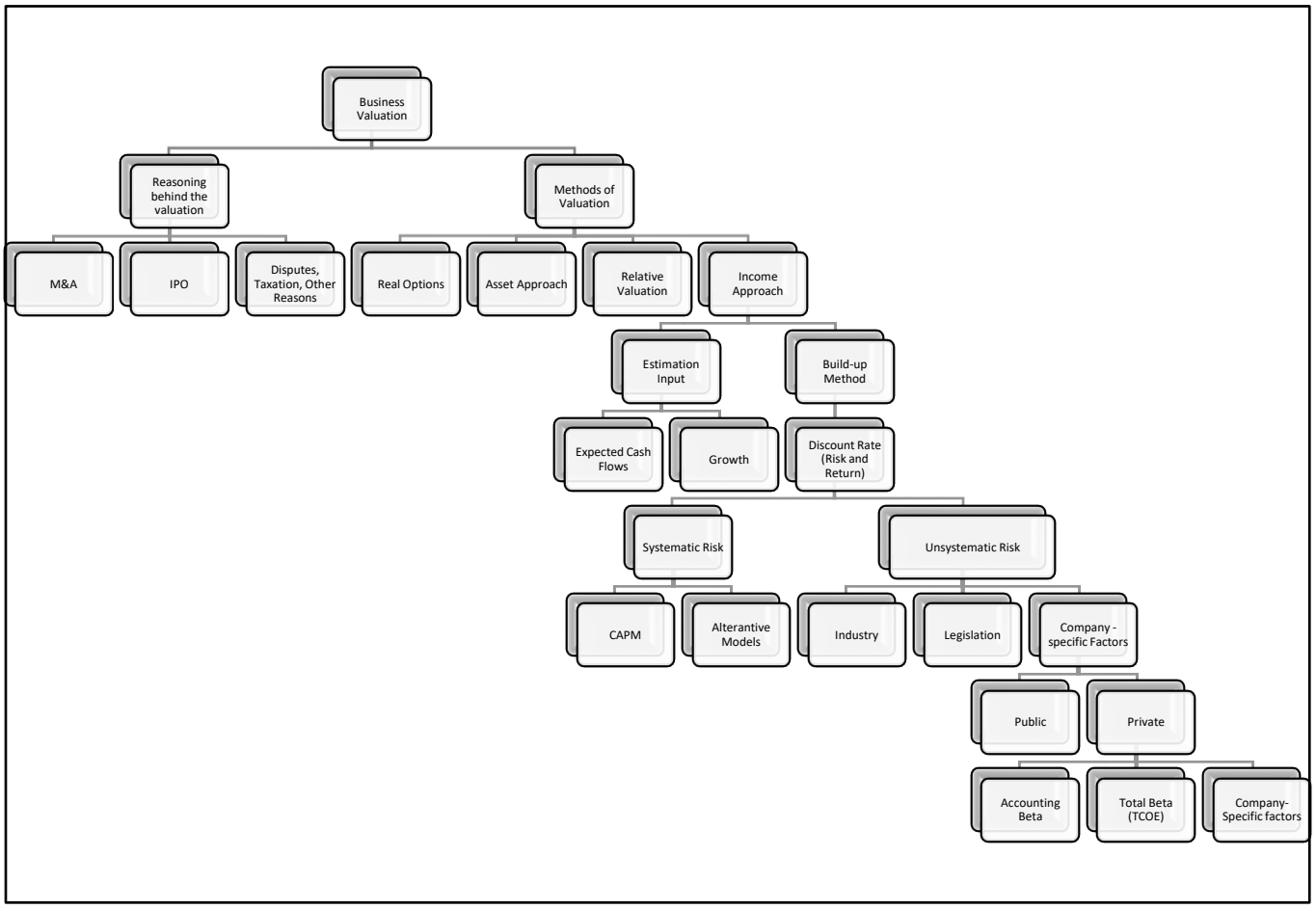


Figure 2: Papineau's Tree

extensive. To provide a good reference framework, we will be covering issues such as the valuation rationale and how the reason for conducting the valuation affects the process. We will also be delving into the methods most commonly applied by appraisers and academics and how basic concepts such as risk and return are incorporated in the input required for the valuation to be conducted. Following this, we will examine how risk is split between systematic and unsystematic, and how it is measured. Finally, we will investigate how all of the above tie with the private companies' valuation, presenting the most recent developments in the field.

2.1 Preface

Business valuation should be viewed as the procedure, or rather the assortment of procedures, employed to determine the financial value of an enterprise. Based on the definition given by the National Association of Certified Valuators and Analysts (2012), business valuation is a method or a series of practices that can be used to set the fair price on which a business could be sold. Determining the value of a company is dependent on a set of principles and hypotheses. Primarily, an appraisal should reflect the reason for which it is conducted, and that rationale is affected in the terminal price. Additionally, that price incorporates the aforementioned hypotheses, which refer to the conditions under which the valuation occurs.

It may sound intuitive as to how one should proceed on exploring a company's worth, however as will be apparent shortly, it is more complicated as different parts come into play, and the process requires a deep understanding of not only the business itself but also the environment it operates in. Furthermore, the valuation is influenced by the reason it is performed. It can be easily noticed that there would be a substantial difference in worth if the valuation was performed by the owner of the business or a third party that was simply looking for an investment target. The value would also be different if the investor expected besides the returns derived from the business, to reap other benefits, for example synergies with other companies they might have invested in. It is sufficient to say at this point that the reasons as to why a valuation is performed range from financial, such as for example a company going public through an IPO, or going

private, to ownership related, in the case of a divorce of a couple that holds the majority of shares in a company, or even in the event of the owner stepping down and allowing their family to take control of the company. The more common occasions, in which a valuation is required are M&A deals and the divestment of an enterprise, due to buyouts or simply with the intention of raising capital, or even in the event of a company's liquidation.

Based on the previous discussion, the idea that value can be recognized at different levels of ownership has emerged. The most typical levels of value can be categorized into controlling, marketable minority and non-marketable minority interests (Mercer, 2008). The marketable minority valuations are positioned in the middle between the other two as they represent the equity that can be freely traded without constraints at any exchange and are usually less than half of the equity that a business has available at any point in time. If an investor wants to have increased control over a company, they will target the upper level of value, the controlling interest. With this they acquire more than half of the company's equity, and subsequently enjoy the benefit of being able to determine the management of the company, its financing structure, the dividend policies and the overall strategy of a company. These levels of control are usually preferred by investors, as noted previously, who seek synergies with their other investments. The final level of value is the non-marketable minority, which is typically the focus in private company transactions, and they are characterized by heavy frictions and overall uncertainty, as there is no available market for them. This illiquidity is one of the main contributors to the higher discounts in the sale of private enterprises.

The next step, after having touched on the topic of the different levels of value, we will now, briefly, examine the tools appraisers have at their disposal in order to determine the worth of a company, namely asset, market (also referred to as Relative valuation) and income approach. The first and perhaps the most conceptually straightforward of the methodologies is the asset approach, in which a business is simply considered as the aggregate sum of the market value of all assets and liabilities it has, which not only encloses the problem of accurately determining what constitutes an asset or a liability, but also the fact that this method does not incorporate the important aspect of future growth. The second method is the market

approach, which focuses on the value of other businesses similar to the one being examined, to benchmark and determine its value. The third approach is the income method, which as Damodaran (2002) argues is the basis for all other methodologies, and is premised on the idea of the value of a company being determined through the expected future income cash flows it will generate for its investors. This methodology requires a variety of inputs with the primary ones being the required rate of return or discount rate and the future forecasted growth rate. An example of this method of firm valuation is the following:

$$Value\ of\ the\ firm = \sum_{t=1}^{t=n} \frac{Expected\ Cash\ Flows\ to\ the\ Firm}{(1 + WACC)^t} \tag{2.1}$$

Although the most commonly used methods are the three we reviewed previously, a more recent valuation method that has been steadily gaining acceptance, especially by academics (see for example Favara, Schroth, and Valta, 2012), the real options approach, or contingent claim. It is tangent to the idea that the all investment decisions can be viewed similarly to the option model developed by Black and Scholes (1972), in the sense that the value of any asset has characteristics similar to that of an option. To explain this further we turn to the reasoning presented by Damodaran (2012) and Penman (2010), who explain that the investment decisions faced by managers can be viewed through the payoff functions of put and call options. If an investor believes, for example, that the asset's value will exceed its current value at a specific point in time, they can choose to exercise their right and invest in the asset. The downside of this method is that it cannot accurately represent the investor's choices when the options have long lifetimes. Also, as these options can be present on items that are not traded and thus the variance cannot be accurately estimated, any calculations done, are more prone to error.

One critical feature in all valuation appraisals is the necessity to determine the appropriate rate of return or discount rate. A method for determining the discount rate, that is primarily used in private businesses' valuation (National Association of Certified Valuators and Analysts, 2012), is the build-up method, which

is primarily linked to the income approach, and its centerpiece idea is that the discount rate used in the valuation is a sum of all the returns required to accept the risks a company faces. This methodology represents the idea that in order to determine the requisite rate of return on an investment, one must examine the macroeconomic environment in the which the company operates, the industry and finally the company's fundamentals to determine the risk the company faces and ultimately determine the required rate of return. The generalized formula of the capitalization rate, as it is proposed by Butler and Pinkerton (2008) is the following:

$$Total\ Cost\ of\ Equity = riskfree\ rate + beta * equity\ risk\ premium + size\ premium + companyspecific\ risk\ premium \quad (2.2)$$

This will be considered in detail in the appropriate section of the review, at this point it is sufficient to say that this method has been gaining acceptance over the recent years mainly due to the implementation of the Total Beta, a measure of the company's risk, proposed by Damodaran (2005b).

The methodologies discussed previously are important, not only for the obvious reason of determining the value of the company, but also because it enables the observation of some key characteristics of the process itself. It implies that a successful valuation requires preparation on behalf of the appraiser. It is important for them to understand the business and what their product is. The valuator also needs to determine what the position of the firm is in the industry they operate within, and how it fares against its competition. Also, at this point, he must understand how the market for the specific product is structured, does it have barriers to its entry, is it an oligopoly, a monopoly or a market where conditions of perfect competition exist. But before all that, the appraiser must examine what the macroeconomic conditions are for the country or countries the company operates in. Factors such as inflation, interest rates, consumer confidence and unemployment rate can have a significant impact on the business cycle of the firm (Fisher and Statman, 2003). All these elements are essential indicators for evaluating how a company will grow.

The environment in which the company operates however is only one of the two determinants of the businesses' fair value. The other is the company itself. Having become familiar with the business and its product, an appraiser must then focus on the forecasting of the company's future financials to determine its potential to generate cash flows and what portion of those can be distributed to the investors (and what portion will be reinvested in the firm to produce future growth). As value is created for investors through the capital that is being distributed to them they should always seek to maximize that amount through their investments, however there are occasions, for example when a significant project needs to be financed which will increase the future growth of the firm, where reinvestment is considered to be desirable (van Binsbergen and Koijen, 2017). For the investors in mature businesses, it is the former case, while the latter holds for investors in growth and young companies.

At this point the appraiser will need to calculate another element, the cost of equity capital (discount rate) or the weighted average cost of capital, depending on whether the equity interests or the total enterprise is being valued. To do that the valuer needs to take several determinants into consideration. We will cover this particular part extensively in the review, so at this point it is sufficient to say, that the several forms of risk faced by the company need to be considered, both in the systematic and the unsystematic aspects of it, namely risks associated not only with the market but ones that are specific to the company. All these risks are what defines the discount rate, or to expand a little more on it the rate of return required by the investor which is a composite of the compensation for the time value of money, inflation and risk, and subsequently are reflected in it. One of the most popular ways to assess market risk has been the CAPM (with the Beta as its primary risk measure), however as we will see other models have been developed in an attempt to address some of the criticisms which have developed.

Company specific or unsystematic risk is also called diversifiable risk as it is that component that can be diversified by investing in several companies. This is not feasible for the investors in private enterprises, because their investors have usually most of their wealth tied to the companies, they are invested in. This complication implies that private companies are linked to higher discounts and all the risks associated with

them need to be calculated for their valuations to be effective and valid (hence the use of the build-up method that sums all forms of risk associated with them).

In addition to the calculations of the input measures directly noted, the valuation needs to include a series of adjustments. The appraiser needs for example to adjust the financial statement elements for inflation[1] or other adjustments targeted to compensate investors for other aspects (for example their time). Besides the adjustments and risk another important factor is the potential for firm growth. That input is extremely significant and although it might seem intuitive at first, in practice it was introduced in the book by Gordon, (1962), in which the Gordon Growth Model (GGM) was introduced, which expressed the current price of a company's stock as a function of the dividends it would produce for its investors in the future, while keeping a constant growth rate for the dividends.

It becomes clear that business valuation is a complex procedure that requires not only a stalwart methodology but also significant insight and intuition from the appraiser. This complexity is what gave birth to the vast literature on expected returns and discount rates. The goal with this review is to present an overview on how the literature has evolved over time, point out the milestones in it, and show the latest developments regarding the valuation process and the discount rate. In order to do that we deemed it important to set up the general framework, on business valuation, the methodologies employed and the reasoning behind it. We will now proceed to explain in greater detail, all of the concepts and ideas developed in the introduction, and thereby explain how this current research study is positioned within the literature and how we will expand on it, through the use of a series of analytical tests.

The rest of the literature review is organized as follows. Firstly, we will explain in detail the reasoning behind the valuation procedure. We will then review the tools appraisers have at their disposal in order to perform it, proceed to define risk and the return investors expect, and look at the two components of the

[1] When analysts review a series of financial statements over a period of years, they need to adjust these elements' value for inflation. This is a standard practice in academic papers also (see for example the paper of Michaely and Roberts (2012) but rather rare in practice.

risk, namely systematic and unsystematic. Both of these constituents will be analyzed extensively, as we will review topics on macroeconomic factors, the CAPM, other models that succeeded the CAPM, the equity market risk premia and of course the supply side models. We then proceed on the unsystematic risk element and review topics related to industry, legislation and company specific factors. We will finally look into the developments regarding the private companies' literature and explain both alternatives to the CAPM Beta but also discuss some factors that are exclusive to private businesses. In summary, the review will provide a clear overview of what has transpired in this field and subsequently an understanding that what we are doing is important.

2.2 Valuation Rationale

Having presented what will be covered throughout this review in the introductory section, we will begin the review by addressing the ideas and reasoning behind the valuation of businesses and by going into these topics in greater detail. To do that, we first need to explain how companies operate and how they generate capital, which can potentially lead to growth and yield the return that investors look for.

In providing a framework for business valuation Mercer (2008), presents a series of constituent principles that he refers to as G.R.A.P.E.S, which is an acronym for growth, risk and reward, alternate investment opportunities, present value of the investment, expectations that the investor has and sanity, rationality and consistency that are the basic characteristics, that will allow them to properly value the investment. These principles reflect the Time Value of Money (TVM) and are embedded with the ideas that investments are expected to produce further value through a series of cash flows they will generate within a specific timeframe. To expand on this, it becomes evident that a valuation is an assessment of the relationships between the primary value factors of risk and return, income generated and growth over time. This makes it clear that the most important element, to properly value any asset, is to understand the differences in the

risk profile of the various asset classes. The risk inherent in every asset is expressed, in its valuation, through the level of the discount rate.

Another notion that Mercer (2008) highlights, is that value is created at different levels[2] within the firm depending on the type and proportion of the control or level the investor desires to have over the company. Mercer separates the various control levels based on the ease of being able to transfer ownership to other investors or not (marketable vs non-marketable). This type of separation also dictates the discount rate or the premium an investor has to pay (or receive) in order to acquire (or sell) their interests in the firm. After the transaction of transferring the control interests to another investor, the new "owner" can realize gains through either strategic voting and financial control[3], or at the level of marketable minority shares, which constitute enterprise levels of value, at which value is achieved through the respective cash flows generated by the equity's cash flows. This major characteristic, that is especially prevalent for private firms, is also incorporated in the discount rate, and accounts for discounts for marketability, various control premiums and minority interest discounts.

[2] In essence, the value of an investment in a firm, should be viewed as the "gains" an investor can realize by tying their wealth to the company they invest in. Those gains can vary depending on the degree of attachment the investor has to the company. This is why we have different stakeholders in a company.

[3] By controlling the management for example or by realizing synergies with other investments these stakeholders might be invested into.

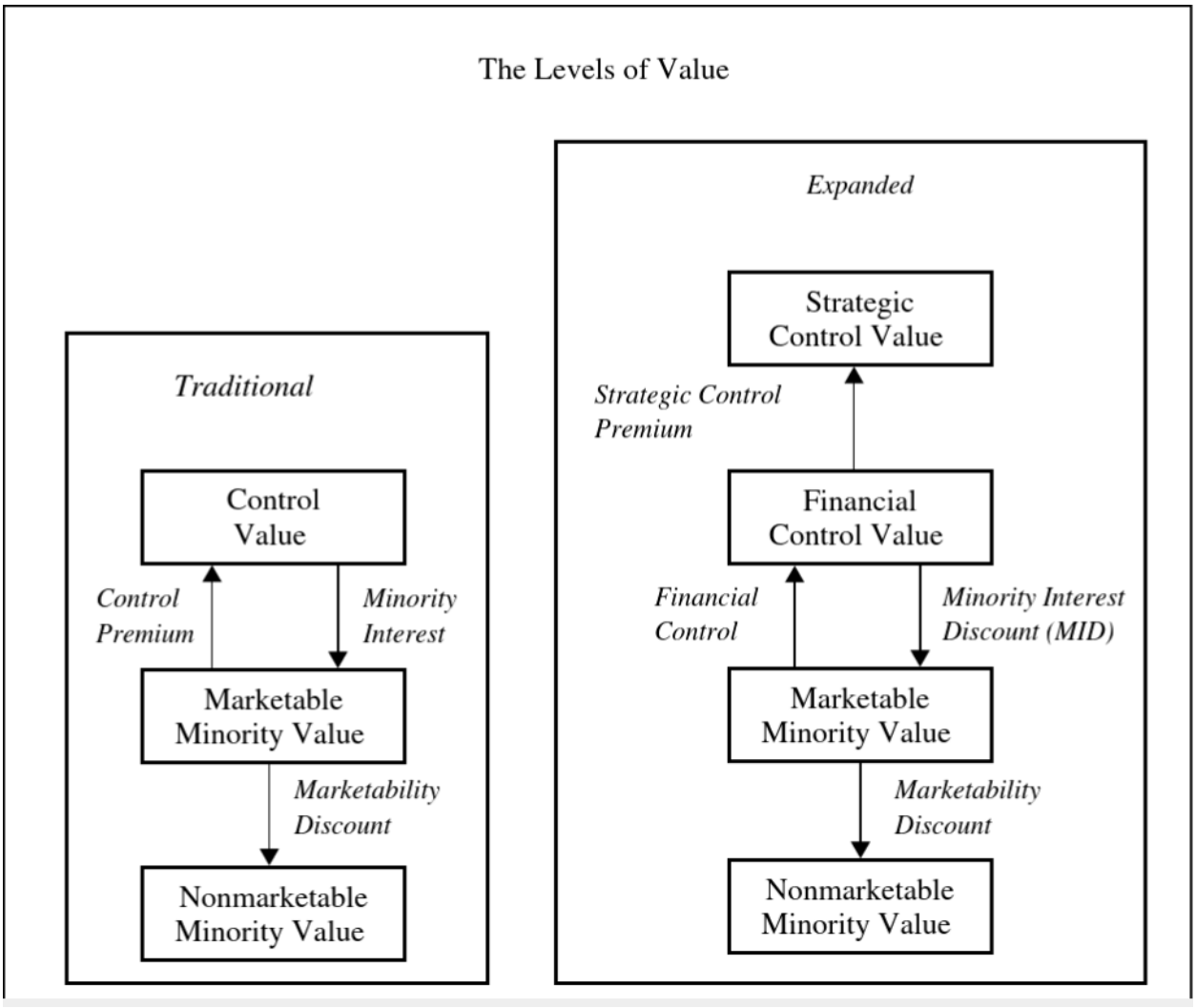


Figure 3: The Levels of Value (Mercer, 2008, Business Valuation: An integrated theory)

For the secondary part, namely to explain how the operations of businesses are affected by investments, we start with the writings on financial analysis by Penman (2010), who states that investment in firms comes through the firm's equity or debt, with investors being divided into debtholders and shareholders. A special category, though, is that of investors who have contingent claims on the company, such as options, convertible bonds, etc. Debtholders provide liquidity to the company through various forms of loans, while shareholders do that by the cash, they subscribe to buy the company's shares in the primary market. Both

types of investment, create a claim on the firm, or to put it more directly an obligation on the company's part to repay its creditors, through a stream of cash flows (payments), for example interest payments or dividends, depending on the type of investment.

Each investment is made to achieve the stream of cash flows that they are expected to generate in the future, and thus it is implied that asset prices will reflect this potential income. The amount of money that the investment will generate, excluding the amount paid to acquire this specific asset, is this asset's return. Investors may also deem appropriate, to sell their claims to other investors, if they think that the returns are not high enough, or they need short term liquidity. This creates the idea of the firm generating value constantly to its investors, through direct and indirect means. This is also reflected by the shareholder value maximization notion, which is one of the pillars of finance and accounting theories, with various implications for both investors and firms. Taking all the above into consideration, one can view the valuation of an enterprise as the sum of the value of all claims on the firm. It is important to stress however, that each firm (or any asset) has its own distinct characteristics, which makes it difficult to put an accurate price on it, even within firms (assets) of the same class. It is logical, though, that no one would be willing to invest more in an asset than what its actual worth is.

To create value, Penman (2010), explains that companies participate in a number of activities which can be broadly categorized into financing, operating and investing. Money invested in firms, are allocated through a series of strategic decisions, made by managers or internal analysts, with the objective of creating projects that will make the company realize synergies or increase production, or market share etc., with the purpose being, to create greater cash flows at an acceptable rate of risk. These types of inquiries and decision-making processes are commonly referred to as value-based management. Investors also use experts and external analysts (with quite a great variety of them, from tax experts and accountants to security and credit ones), to determine the worth of a business and the projects it pursues. All those experts' opinions are based on quantitative programs and analyses, which vary greatly among them, mainly due to a series of reasons

that will be explored further in the study that follows. One thing that it is certain is that as there are many different aspects and people involved in the valuation process the more complex this process becomes.

It is important to also point to the significant work that has been done by Damodaran (2012), who is considered one of the leading experts in the valuation field, who attempts to clarify and more clearly explicate some of the common misconceptions regarding valuation. The first and perhaps the most important one is that, valuation is based strictly on quantitative analysis and thus the estimates, that are a product of the process are absolute. It will be apparent as this study progresses that regardless of the methodology employed to determine an asset's value, there are a series of underlying assumptions which are subject to appraiser bias. For instance, due to the different levels of information that analysts have access to or based on their cognitive bias prior to performing the valuation of the stock, or even more commonly by succumbing to managers' pressure, they may be more prone to issue a buy order on the stock. Another common misconception is focused on the time frame during which a valuation is valid. As the process is tied to a series of firm related information, that are valid only at a specific point in time, one can easily deduce that a valuation has merit only up to the point that it was performed, if all available information up to that point was taken into consideration.

It becomes apparent that the main idea behind the valuation process, is the determination of the actual worth of a business. This task however can be subjected to many issues especially since each case has distinct characteristics that separate it (even if only marginally) from the others. With that said, though one cannot help but notice that the underlying principles that govern the valuation are similar. In fact, ideas such as risk and growth are present always, in all investment activities. Those are what determine value in the end, and as such their study can shed some light, or even allow researchers to create a framework that will provide them with a set of guidelines as to how to accurately define fair value[4].

[4] In this occasion with the term "fair value" we refer to the intrinsic value of a business, which is the monetary worth of the company, to an investor who is rational and has all the information available about the company.

2.3 Methods of Valuation

Having explained what the main ideas behind any valuation are, we can move on to reviewing the tools that appraisers have in their disposal as means to determine the value of different assets. Analysts have a great variety of models to choose from, when performing valuations. Damodaran (2012), argues that despite the differences across the various methodologies used to perform the valuation of assets, those techniques present more similarities than differences. Furthermore, he suggests that the principle of parsimony is important, when applying a methodology to arrive at an asset's value, contrary to the popular opinion, that more complex models produce more accurate results. He attests to that by pointing to the importance of the analysts' ability to clear the useful from the non-useful information and construct an appropriate model based on that. He concludes that an accurate valuation will include several factors that affect the value of the company (or the asset in question), rather than accepting on face value the results that standard valuation models will produce.

Regarding the methodologies usually employed, these can be classified into three distinct categories. The first, and probably the most commonly used, is that of relative valuation, which estimates an asset's value through the prism of several variables of assets with similar characteristics. The relative valuation method or multiples approach can be roughly divided into two separate groups, that of comparables and that of multiple screenings. For the method of comparables, the user needs to identify several enterprises with a similar business to that of the target company, determine which variables better reflect the company's abilities to generate cash flows (for example earnings, book value, etc.), get an estimate for the multiples of those variables, and finally apply some form of average on those estimates to get the value of the firm itself. According to Penman (2010), this valuation method is most commonly used, due to its simplicity, especially in the case of valuing private firms, since there is a lack of available information regarding those firms, so comparing them with their public counterparts provides a quick and tractable solution. However, this method is bound to a series of problems, mainly due to the original assumptions on how these variables

should be constructed. Specifically, even firms within the same industry, exhibit a great deal of differences in their size, growth potential, market of operations, leverage and ultimately in their risk profiles.

Similarly, the alternative relative technique of screening, is most commonly used for buying and selling stocks. Under this methodology, a multiple is chosen (good examples of that would be the Price to Earnings (P/E) or Price to Book (P/B) ratio) and in addition to stock ranking is primarily used in the valuation of firms that are going public (see for example Lee and Masulis, 2011). Based on that multiple, stocks are ranked from highest to lowest and an investment strategy is created to determine which stock to sell and which to buy. Yet again, this method is easily done and requires a small amount of information but ignores several factors. Specifically, this method is based upon previous returns, in an attempt to predict the future ones, something that is not easily done, since it pushes investors to underestimate the risk they undertake when they invest in specific companies. This could eventually lead to them realizing losses. Trading on a small amount of information, also leads to the danger of being outwitted by other investors who have been doing more thorough research into a company and better understands how the operating cycle is, for example, affected by the various macroeconomic factors, or how undertaking a specific project might affect its capabilities of generating cash flows.

The second methodology that is used by appraisers, is the Discounted Free Cash Flow, which is the most prominent portion of the income approach of valuation. This methodology is extremely historically significant and to exemplify that one can refer to the use of the Discounted Cash Flow (DCF) methodology application in the Tyneside coal industry in the early 1800s. Studies such as the one by Brackenborough, Mclean, and Oldroyd (2001), who explored the birth (or rebirth as the authors explain) of the discounted cash flow models and their implementation in the industrial revolution in the United Kingdom, explain the reasons that this type of valuation methods came into the forefront. Specifically, the authors suggest that the adoption of DCF models not only during the industrial revolution in the Tyneside coal industry but even earlier than that, is a testament to the necessity to link the risk associated with an investment, to its potential returns, as the surveyors assumed the role of both the cost accountant and that of the appraiser. Using an

extensive sample from archived viewer records for the years 1700 to 1820, they find that, among other things, the DCF methodology was used as a response to the increase in earnings options and the cost associated with investing in the coal industry.

According to Damodaran (2012), DCF is the basis for all other methodologies, as the principles that rule it are universal. Similarly, Steiger (2008) examines the theoretical and practical forms of the discounted cash flow model. His view supports the notion presented before by Damodaran, however he concludes that, although DCF is a compelling way of conducting valuations, it may be subject to assumption bias. Despite its problems though this methodology is considered as the most rigorous, valid and informative by many academics and practitioners. This methodology uses the present value theory, a direct reflection of the time value of money, to determine the value of any asset's cash flows in the future, discounted by an appropriate discount rate, which is representative of the risk adjusted return related to this asset. This model has many variations, however the basic idea remains the same, whether it is used to find the value of equity or the value of a firm. The other methodologies still use the ideas and elements of this one, in both determining the expected outcome and the risk of an investment.

The final methodology employed as standard by valuators, is the asset-based approach, which is probably the simplest and most straightforward of the three. Under this method, the value of a firm is determined as the sum of all its assets minus the liabilities at their current worth. Although it is easy to implement, this methodology has a serious flaw. It does not account for the expected income that investing in a firm will produce, neither for other potential synergies that might arise, but does provide a low value base as a value anchor for comparison with other approaches. Moreover, it leaves out the element of risk, which is what investors need to be compensated for when tying their wealth to an investment. We can see that the constituent elements of a valuation are an appraised insight to future cash flows and an assessment of an appropriate rate of return. All other processes hinge on these two critical elements.

Build-up Method

After reviewing the most common methodologies in the field, it is important to explain at this point a method that is frequently employed by appraisers, especially on the valuation of private enterprises. The Build-up method is used in the estimation of the after-tax net cash flow discount rates. It is essentially the total of all the risks inherent with investing in a company (Butler, 2010) and its roots can be traced in the fundamental principle of greater reward required for higher risk associated with a particular investment (which will be analyzed further in the section that follows). The discount rate, which is the final product of this method, is created through the use of various components, as shown in the formula 2.3, as it is explained in National Association of Certified Valuators and Analysts (2012) chapter 5 (p.7) on capitalization and discount rates, that follows:

$$K_e = R_f + ERP + IRP_i + SP + SCR \tag{2.3}$$

Where K_e is the cost of equity, R_f represents the risk-free rate, ERP depicts the expected equity risk premium, IRP_i is the risk premium for the industry the company operates in, SP is the size premium, and finally SCR, the company's specific risk.

The cost of capital, depicted in the discount rate of an asset, reflects the opportunity costs, which investors require for investing their money in a specific company or project instead of spending them in an alternate investment. The cost of capital depicts the risk, and as a consequence, the riskier the investment, the higher the reward needed to attract potential investors. A good way to counter the risks of investing in a firm (or an asset), is to do the intuitive thing and invest in many different firms. Creating a well-diversified portfolio, reduces the danger of realizing high losses. This is achieved by simply minimizing the overall danger of losing all of one's wealth, through holding assets with various degrees of risk, that cancel each other out by being negatively related in volatility.

The first step is determining a risk-free rate, for which the rate of return from the long-term government bonds[5] is one of the measures most commonly used (Chen, 1991). Investing in stocks however is arguably associated to a higher degree of risk than government bonds and subsequently a premium has to be added to the required return, which is the equity risk premium. The next component that needs to be taken into consideration at this point is the size of the company (small or large capitalization), which is a well-established factor throughout the literature but also in the industry (Israel, 2011). All these variables are incorporated in what constitutes the systematic risk of the final discount rate.

To those the unsystematic part of the risk needs to be added. This can be decomposed to industry risk, which is related to the risks associated with all the companies that are part of a specific industry and the risks (or rather risk as the total of all the risks) associated with the company that is under appraisal. The latter part of unsystematic risk has not been researched adequately until recently, when Damodaran proposed Total Beta (Damodaran, 2005), as a measure of relative volatility ratio of the company, that reflects the specific risk associated with a firm. We will expand extensively on this concept at a later chapter, as it is a key variable in this research, so at this point it is sufficient to view this measure as the company specific risk measure that can be used in the build-up method.

The pattern of the summation of the various risks to produce the discount rate for the valuation of private businesses will emerge several times throughout both the literature and this thesis. It is important to remember that a private company has several other "burdens" that affect it. As we have seen already, private businesses' investors face risks that cannot be reduced through diversification and the small size (or capitalization) of these companies is a concerning factor for them (we mention small cap as the norm for private businesses as most of them are predominately SMEs as Abudy et al. (2016) explain). The discount rate, for private enterprises, is also affected by the controlling interest levels we presented previously, as private companies are potentially represented through a number of non-marketable, minority levels of

[5] The risk-free rate is compensation for the time value of money and inflation expectations. In addition, academics use treasury bill i.e. cost of three months money and appraisers use longer term government bonds anywhere between 15 and 30 years as this from their perspective covers the average life of a company.

control, which contribute to the discount of the company's value, as those cannot be traded easily (Comment, 2010) and therefore make transactions regarding private businesses' interest less liquid than those of their public counterparts.

As we have set the groundwork for the ideas of value and how it is created it is time to visit the risk portion of the valuation process. In the following section we will be examining the risk and return relationship and present a comprehensive framework on how the related literature has evolved over the years.

2.4 Risk and Return

Companies can raise the capital needed to fund their operations and pursue any investment project from two sources. They can either finance their activities with debt (in the form of loans, bonds and overdrafts among other instruments) or they can turn to their shareholders and raise funds through equity. In the case of debt, the company promises to make payments of interest to the debt holders at the predetermined contract dates, until the debt matures, at which time the principal amount will also be repaid. For equity on the other hand, the company issues shares, that represent a claim on the value of the firm, however there is no guarantee that regular dividend payments will be made, and the capital claim on the company's value can be exercised after any outstanding debt has been repaid or by selling in the secondary market.

We have referred to the relationship between risk and return many times throughout the review already, as it is this relationship that defines the final outcome of the valuation process, which is the expected return that is required by investors. Investors are fundamentally considered as risk averse by most of the accounting and financial theories (Sharpe, Alexander, and Bailey, 1999), and for that reason, to compensate them for accepting higher risk they should be rewarded with higher returns (Brealey and Myers, 2003). This notion is a determining factor in the financing of enterprises and has spurred a considerable amount of academic research on the topic, which will be further analyzed in the following sections.

At this point it is sufficient to say, that investors are faced with two types of risk in an investment, diversifiable (unsystematic) and non-diversifiable (systematic), and they should only account for the latter. This idea is the centerpiece of the largest part of the discount rate literature and it is best expressed by the Capital Asset Pricing Model (CAPM), which we will explain in some detail in the chapters that follow. Jorion (2000) puts this intuitive idea into context, when he states that publicly listed firms are a means for individual investors to spread the risk of ownership in a company across the market. He emphasizes though that this idea does not hold true for investing in private enterprises. To understand why, one simply has to think about the inability of investors in such firms, to properly diversify their portfolio, as their wealth is usually tied to their firm, and therefore they lack the capacity to spread their risk. This is particularly important within the premises of this thesis as the main focus is on private enterprises, and as such it will come up quite often.

2.4.1 Systematic Risk

We will begin the analysis of the risk that investors face with the examination of its systematic part first[6]. Systematic risk is the uncertainty associated with the macro economy whether local, national or global that a company operates in (for example changes in the government interest rates, oil prices, etc.) and is the part of the total risk that cannot be diversified away by investors (Chen, 2003; Cooper and Priestley, 2016; Demirer and Jategaonkar, 2013; Miao and Wang, 2007). This part of the total risk is important regardless of whether we are examining stocks in a stand-alone setting or as part of a portfolio, as some stocks seem to be highly correlated with overall market returns, and thusly are more prone to changes in the market. Capital market theory suggests that the standard deviation, which in essence measures an equity's systematic and unsystematic risk, is what determines the equity's expected returns (Markowitz, 1991). It is no wonder that Lintner (1965a) suggests, explaining the significance of diversification, that an investor

[6] To enable a clearer view of the literature and how it is structured, Figure 4 presented below, summarizes the ideas that are covered in this section of the review.

should focus on the correlation between the stocks and the market, as value for them derives mainly from it, but also, from the from the fact that correlations between the various stocks in a portfolio are not perfect. This notion is particularly important in modern portfolio theory, as all the studies that focus on systematic risk, suggest that unsystematic risk is not compensated in equilibrium as it can be diversified away, and essentially that leaves systematic risk as the major determinant of a stock's returns[7].

Several studies are dedicated to determining the factors that influence stock prices, or rather the link between individual stocks and the market, and how investors are rewarded for accepting the additional risk, in excess of the risk-free return, also known as the equity market risk premium. The literature on this topic, branches out into several different streams, that can be broadly categorized by the factors they focus on, in their attempt to measure systematic risk, and they range from macroeconomic (Cooper and Priestley, 2009) to behavioral (Kothari, Lewellen, and Warner, 2006) and financial ones (Ryan, 1997). Another part of it concentrates on the measurement of this portion of the overall risk, with the CAPM, and all the literature developed around it, being the most prominent representative of this category.

[7] However, we need to remember that this is rarely the case for private businesses, a topic which we will cover at length in following sections.

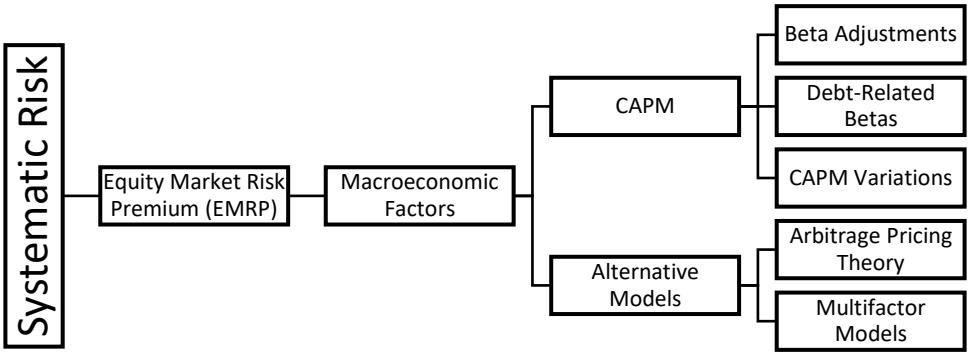


Figure 4: Systematic Risk

The most well-represented category, that has been researched, is that of the macroeconomic variables. A great number of papers have explored the relationship between stock prices and how those are tied to the overall economy. For instance, Chen, Roll, and Ross (1986) argue that there are several macroeconomic variables that significantly affect stock prices, such as long and short-term interest yield curve, inflation, production volume, as well as the yield of bonds. They state however, that although there is a general agreement as to the importance of systematic variables, in essence the variables themselves and their connection to each other, have not been adequately explored. Tabner (2012), also argued that besides the traditionally accepted macroeconomic variables, the size of the companies that are part of an index, as this is expressed through their market capitalization, also affects the premia investors require. In particular, he explored the effect that systematic risk in conjunction with capitalization weights have, upon the variance and returns of the FTSE 100 Index, and his findings suggest that systematic risk does not have a significant effect upon the index itself, as the firms that are included in it, are large enough to have lower than average covariance. Some examples of macroeconomic factors can be found in Table 1 below.

Macroeconomic Variables that affect stock returns
Spread between short- and long-term interest rates
Expected and unexpected inflation
Industrial Production
Spread between high and low-grade bonds
Consumption
Production Technologies
Fundamental Shocks (e.g. recessions)

Table 1: Examples of Macroeconomic Variables that affect returns according to Chen, Roll and Ross (1986) and Cochrane (2011)

Several other factors have been suggested as determinants of the systematic risk. A significant portion of the papers on market risk associate it with earnings, as well as several operating risk factors and operating leverage[8] (Ryan, 1997). Corporations seem to respond to that by lowering their financial leverage exposure, as a means to reduce a level of the systematic risk they face, clearly indicating the relationship between market risk and the level of leverage. It is also pointed out, that a greater level of reporting and disclosure in the form of fair value accounting, combined with the adoption of direct costing and higher segment reporting, would result in better understanding of the operating risks faced by firms, and potentially reduce the effects on them.

A significant portion of the systematic risk section of the literature review will be occupied by the different methodologies that were developed in order to establish a good measure of systematic risk. The most important one, and to date the most accepted method, is the Capital Asset Pricing Model, which was originally developed by Lintner (1965a), (1965b), Mossin (1966), Treynor (1961) and Sharpe (1964). This model introduced the Beta, which is a measure of a stock's systematic risk. As we will see, it has been one of the most controversial concepts to date, and gave birth to several other methodologies, that range from

[8] A company's fixed cost base, and thereby earnings, will be more greatly impacted by systematic risk when a company has high fixed cost and low variable costs.

iterations of the original (Liu, 2006) to others that completely refute it and suggest other measures in its place such as the Arbitrage Pricing Theory (Ross, 1982) and Multifactor models (Fama and French, 1993).

Proponents of the CAPM (for example Fletcher (1997) or the more recent paper of Brotherson, Eades, Harris, and Higgins (2013)) argue that its popularity is based on its simplicity but also on its accuracy to provide good estimates. Others, such as Chen (2003) and Santos and Veronesi (2006), accept its effectiveness in explaining systematic risk, however they propose adjustments to it to counteract the issues its critics have identified (the fact that historical returns data are used in its estimation, not including an adjustment to account for the small size of firms, or the use of a major market index to estimate the market return for example as these previous two studies suggest). Another prominent point in the literature is that a company's Beta signifies not only the risks of its projects but also is an indicator of its financial structure. This idea generated the different versions of Beta. For example criticism on the CAPM came from the early studies of Blume (1971), who proposed a simple adjustment to account for the fact that Betas tend to move over time towards the market average of 1, and Vasicek (1973), that introduced a Bayesian adjustment to the Beta, by utilizing the standard error.

The systematic risk part of the literature is probably the most extended, as its related research began over seventy years ago and is still ever-growing. In order to be as thorough as possible, we will begin the analysis by explaining the equity market risk premia and the supply side models that were developed in order to measure them. We will then proceed to overview the corpus of literature that focuses on the CAPM, by explaining its development in detail and providing some insight on the controversy it created over the appropriateness of the Beta as a measure of a stock's systematic risk. We will also review all the models that were developed as a response to the CAPM's shortcomings. Finally, we will review the macroeconomic factors that have been identified to be related to systematic risk.

2.4.1.1 Equity Market Risk Premium (EMRP)

One of the most important literature sub-sections in the systematic risk corpus, is the Equity Market Risk Premium (EMRP). The EMRP, as cited in a number of papers (Avramov and Chordia, 2006; Bali, Cakici, and Chabi-Yo, 2015; Bartholdy and Peare, 2003; Gallagher and Pinnuck, 2006), is the additional reward investors require for the systematic risk they accept, in order for them to participate in a particular investment, instead of investing in the risk-free instrument. Or as Aggarwal and Goodell (2011) explain the equity premia are the indicators of the additional compensation equity investors require to provide firms with capital, as well as, for them to be able to develop plans on meeting their long-term commitments. It is, according to them related to a series of indicators that differ among countries, with larger ones, with well-established financial structures, better governance and higher wealth being associated with lower premia, pointing to an inverse relationship between these factors and the EMRP. The same idea is also supported by Guo (2011), who suggested that underpricing in an IPO (the difference between the offer and the first-day closing price) reflects investors' perceived danger for a company. To support his argument, the author explains, that if investors were given a constant discount rate, underpricing could be analyzed into two constituents. The first would be a constant, reflected by the constant rate given and the second would be the perceived systematic risk, as this is reflected by the equity risk premia.

Besides the macroeconomic factors, the interlinkages between the various markets has been explored, as an explanation for the equity risk premia. Chan, Karolyi, and Stulz (1992), for example, argue that one of the major factors that affects the risk premia on US assets is the effect of foreign capital markets, mainly due to global market integration. As they explore the link between US markets with other international ones, they discover that the domestic returns are highly correlated to foreign market returns, and more specifically to those of the Japanese Nikkei 225 index, but not to their own lagged returns. Similar findings come up, when the MSCI Europe Australia and Far East (EAFE) index is used, and those findings hold even when different econometric approaches are utilized, and this is a solid indication that global market integration is a determinant of the risk premiums.

The link between macro focused factors and the EMRP has however not been universally accepted. For instance, Lamont (2000) attributes the market premia not to taxes, interest rates or other country related factors, but he suggests that they are primarily related to firm specific future investment plans, or in simpler terms, the uncertainty associated with future projects on behalf of the companies is what fuels the increase in the premia. He also finds that the premia (which he proxies as the discount rates in company transactions) should be viewed as time-varying, as companies decide to shift their plans for future investments over-time. These findings are further verified by the Cochrane (2008) study.

Another approach on whether macroeconomic variables are appropriate in the calculation of EMRPs is that of Neely, Rapach, Tu, and Zhou (2014), who follow a somewhat different route and attempt to relate the prediction of equity risks, with the technical indicators used by practitioners, by comparing the results produced by models based on macroeconomic variables, and those from technical factors[9]. The results point to technical analysis displaying the same explanatory power as macroeconomic variables do, with both types overcoming each other's weaknesses, when explaining the business cycle. Technical indicators exhibit an ability to forecast the declines in EMRP when the cycles approach their peaks, while macroeconomic determinants are related to the increase in the EMRP. Both of these types of determinants can result in more accurate estimates, as the movement in the premiums is captured by them.

The major controversy on the EMRP however, can be traced primarily on the methodological procedure that is used for them to be estimated, and specifically whether or not it is realistic to use constant premium or time-varying models. The consensus is that, as Ogier, Rugman, and Spicer (2004) explain, calculating the EMRP depends on the methodology used to do so. To that end a plethora of estimation methodologies have been proposed, ranging from the very simplistic to the highly sophisticated, with historic and forward-looking approaches being the most prevalent ones. Earlier studies, such as that of Indro and Lee (1997), exemplify the simplicity of the original approaches to the topic. The initial studies focused on arithmetic and geometric averages of single periods which, as we will see in later studies such as that of Cochrane

[9] Some examples of technical indicators would be momentum, moving averages and volume of trading.

(2011), lead to the exclusion of the multidimensional nature of returns. Indro and Lee (1997) attempt to determine the biases that impact the arithmetic and geometric averages of single period returns, in order to better understand the variation of long-run expected return yields, that comprise both a stationary and a time-varying component and their results indicate that a horizon-weighted average is a better estimate for the long run stock returns. This is because they record a negative autocorrelation, in the long run stock returns that is enhanced by the use of arithmetic and geometric averages of returns and risk premia. As they report, failure to take those into consideration might lead to misleading results, and subsequently to misinformed investment decisions.

The focus of the literature on the methodological approaches of risk premia is also highlighted in the work of Damodaran (1999a, 2009, 2014), who notes the lack of a good measure for them (which is the beginning point of almost every research paper in this area). He explains that the prevailing methodology for estimating risk premiums is based on historical returns, with the risk premium being the difference between stock and bond returns. However, the main fallacies of this approach are data availability, or the lack of, as well as their volatility.

Damodaran's conclusions are also consistent with the work of Campbell (2007), who states that determining an appropriate equity premium has been the centerpiece of the evolution of the various asset pricing theories, and has undergone several transformations. From being viewed as a constant, to the point where the abundance of available data led to a more accurate estimation of the premium. These findings are contradictory to the results of various studies from the early 1980s, moving from the idea that equity premiums are constant and point to the fact that they are a time-varying variable and are better tied to market inefficiencies. Research in the following decade, followed this trend and still used valuation ratios (for example, P/E, P/B, etc.) to infer the course of future stock returns, although several voices expressed their opposition to that approach, mainly from econometricians, who viewed this type of approach as risky, because of the various statistical inaccuracies that occur. Moreover, those concerns were reinforced by the fact that at the end of the century valuation ratios were so low, this led to negative equity premiums. The

major point that Campbell raises is that financial theory needs to be used in order to accurately define and reduce the parameters used to calculate the equity risk premium, so that only the most impactful are included in the estimations, as opposed to just blindly focusing on the empirical part of the research.

As an adequate methodological framework could not be created, as is apparent from the plethora of methodologies in the field, the horizon over which the EMRPs should be calculated became the centerpiece of the literature. With proponents of both constant and time-varying premia reporting results that back their views. Zhu (2015) is an advocate for the time-varying camp, as he indicates that the finance literature views stock returns as best described by time-varying models, mainly due to cyclicality, different risk aversion levels, rare events and other economic abnormalities. A similar idea can be applied in dividend growth predictability. His research produces the following conclusions, firstly there is a strong time-varying relationship between dividend growth and stock returns. However, the predictability of the returns is highly related, with an inverse relationship, to the volatility levels of the stock market whereas dividend growth, displays high levels of predictability in highly volatile time periods.

Other studies have also given their support towards the time-varying framework. For example, Adrian, Crump, and Moench (2015) propose a new methodology that incorporates time-varying estimations of risk, with regression based estimators for dynamic asset pricing models. They categorize their variables into three distinct elements. The first one refers to those components with nonzero Betas throughout the returns. The second deals with those variables that explain the variation of returns through time and the last category includes both aforementioned ones. Their results indicate that the estimators obtained by this methodology, are similar to those from Fama and MacBeth (1973), when variables are uncorrelated, and Betas can be constant over time. They also find that generalized method of moments and minimum distance methodologies provide similar results. In general, there are several studies that support the idea that risk premia behave in a more time-varying manner than them being constant.

In contrast to the results reported previously, there are studies such as the one by Pettenuzzo, Timmermann, and Valkanov (2014), who point out how deficient the current models used for predicting equity risk premia

are. They explain that although the time-varying return models have been the norm thus far, they produce worse results than models that use constant risk premia. In their study, they suggested a new methodology that incorporates all information available in the market, which should be sufficient to attribute the appropriate premium to a certain level of risk. This leads them to the conclusion that it is possible to rule out the effect of outliers and thus successfully reduce the size of the equity premiums, and subsequently improving the return predictability.

As one can easily notice there are two major points that can be extracted from the literature on the EMRPs. The first one is that regardless of whether we examine earlier or more recent studies, the recurring theme is the need for a better developed methodological framework for the estimation of the premia. The second part is more important however, as the literature has clearly shifted from a single - period towards a multi-period estimation, or to put it differently over a longer horizon. This is particularly important, as we will be examining the CAPM, which was originally developed as a single-period model. The discussion on the effectiveness of the CAPM is basically a reflection of the EMRP calculation debate, not only due to its single-period nature, but also because the various determinants that the original CAPM seemingly ignored, gave birth to a series of methodologies and models. Some, such as the APT and the Fama-French factor model being more promising in the explanation of the premia variation (Cochrane, 2011).

In the next section we will be covering the CAPM, not only on its first iteration but also on the various other forms that were developed over the years to address the original's shortcomings, such as the Blume's and Vasicek's amended versions of the CAPM's Beta, the levered and unlevered, the accounting and the fundamental Betas among others. We will also be exploring the arguments that were presented against it, and how those arguments lead researchers in the development of other theoretical frameworks, such as the multifactor models (which act as the basis for this thesis also).

2.4.1.2 CAPM

The risk associated with an investment and how an investor can shield themselves against exposure to it, is one of most important concepts in finance. Following that idea, an investor needs only to protect themselves against the undiversifiable part of risk, namely the systematic element. For that reason, the Capital Asset Pricing Model (CAPM) was introduced based on the works of Lintner (1965b, 1965a), Mossin (1966), Sharpe (1964), Treynor (1961). It combines the risk-free rate (R_f), usually expressed as the treasury bill, or other government bonds, the Beta of the investment (β_A), as well as the market return (R_m) less the risk-free rate, that is the extra reward that an investor requires to participate in that investment, as can be seen in the formula presented below.

$$R_A = R_f + \beta_A(R_m - R_f) \tag{2.4}$$

One can easily conclude from the above, that the cost of equity (denoted by the R_A in the formula above), is not the same among different investments, as those exhibit different levels of systematic risk.

Regardless, it is a model widely used by academics and practitioners alike. The study of Brotherson et al. (2013) for example, suggests that the CAPM is the highest regarded methodology amongst practitioners. The survey they conducted, revealed that the cost of equity is usually estimated through WACC and the CAPM Beta, however for the Beta they conclude that there is a lack of uniformity in several aspects of the data used to calculate it. This point is one of the mostly criticized aspects of the CAPM. For example, the authors in this research paper explain that although historical data are used in the calculation of the Beta, the results may vary as different time periods can be used (daily, weekly, monthly) as well as the length of time in years used for the regression calculation itself, which might increase or decrease the variance within the sample.

In order to set the basis for the theoretical framework of the CAPM, we turn to Sharpe (1964), who argued that at that time there was no definitive theory that could explain how risks associated with an investment

could be accurately measured. He explains that following the capital market line, the investor has two options, either invest at the pure interest rate or take the price of risk, meaning the extra compensation he will receive for every additional unit of risk he chooses to accept. He then continues that there is no theory that unifies the risk profile of an asset together with the preferences of the investor, and subsequently no meaning can be given to the relationship of the price of the asset and its risk. With that paper Sharpe, introduced a model, to develop a market equilibrium theory, connecting the risk and the price of an asset, and ultimately introduced the term systematic risk in order to explain the undiversifiable risk associated with a stock.

Following that idea, Lintner (1965a) states that the equilibrium values for the risk inherent in assets, are linearly related to their respective returns, variances and co-variances. The variance of the asset's return, which represents its risk, is then added to the covariance of its returns' and the other securities', to create an estimate of the total risk of a security. He argues that the measure of value of an individual security within a portfolio is given at any time by its return's variance and covariance with other assets within this portfolio. He concludes that these findings are consistent with the theory that suggests that investors are risk-averse, value maximizing individuals, and more importantly investment decisions made by these investors are normally distributed, for as long as the investors hold a risk-free asset in their portfolios. Furthermore, Lintner (1965b), extends upon his previous research, by proposing a solution to the optimizing security portfolios by risk averse investors, who can choose between investing in the risk-free asset and short selling. Additionally, he focuses on the assets held by risk-averse investors and establishes the premises for achieving equilibrium, under various combinations of assets. Finally, he provides evidence of the connection between required rate of return and risk parameters.

Simultaneously with those studies, Mossin (1966) published his paper, explaining that at that time there have been several studies on optimal portfolio creation, in terms of risk and reward. The investor's decisions are an amalgamation of value maximization and funding constraints. He then created a model that had exchange of assets as its main characteristic, and subsequently attempted to determine which prices would

satisfy all potential investors, so that the market can reach an equilibrium. Following the previous work on CAPM, Markowitz (1991), explained that his original work on portfolio theory, is concerned with what the behavior of a rational investor would be, in terms of utility optimization. He then compares his work to that of Sharpe and Lintner, and explained that the one complements the other, in the sense that their work shows how an economic equilibrium can be achieved by rational investors. He distinguishes portfolio theory over other theories and focuses mainly on the uncertainty that is inherent to investor behavior.

2.4.1.3 Support and Criticism for the CAPM

The CAPM, although being the most accepted and applied model by both academics and practitioners alike has been rigorously scrutinized. The criticism focuses on a large array of issues, highlighted in several papers that span from its inception till more recent years (Blume, 1971; Chen, 2003; Fernandez, 2006; Hamada, 1972; Piazzesi, Schneider, and Tuzel, 2007; Vasicek, 1973). Firstly, the CAPM is heavily based on the accurate measurement of the sensitivity of the asset's price to the market, as it is expressed through the Beta. It is not clear however, which is the correct type of returns to be used, when the Beta is estimated, with the various databases using different timeframes to calculate it (several examples of the estimations that databases use can be seen in the Table 2 that follows). The next point that is often mentioned is the fact that the CAPM uses historical data (ex-ante approach) to estimate future returns (ex-post), which creates a gap between the expected returns and the actual returns. Moreover, the CAPM is a single-period model, which is unrealistic to assume in the real world. Finally, changing the market portfolio (changing the market) used as a proxy for the complete market produces different Beta results.

	Market Proxy	*Period and Frequency of Data*	*Adjustment Factors*	*Beta for Bristol Meyers Squibb (2002)*
Bloomberg	Over 20 domestic series	Daily, weekly, monthly or annually	(0.67 x unadjusted Beta) + (0.33 x 1.0)	1.05
Compustat	S & P 500	5 years, monthly	None	1.20
Ibbotson	S & P 500	5 years, monthly	Adjusted towards peer group Beta weighted by statistical significance (Vasicek adjustment[10])	1.04
Merrill Lynch	S & P 500	5 years, monthly	0.33743 + 0.66257 x (unadjusted Beta)	1.14
Value Line	NYSE Composite	5 years, weekly	0.35 + 0.67 x (unadjusted Beta)	0.95
Hemscott	FTSE All Share	2 years, monthly	None	N/A

Table 2: Popular sources of data and the adjustments they use to estimate Beta (taken from Mr. Trafford's lecture notes)

Some of the above points are noted in a study by Fama and French (1997), who argue that estimation for the cost of equity are burdened with three major problems. Firstly, the selection of the model is of major importance, with models such as the CAPM not producing accurate results, mainly due to the lack of precise estimates of risk weightings, as they argue that the over-time variability of risk loadings leads to understated estimates of the cost of equity. The imprecision in factor risk premiums is also another major problem. The CAPM for instance estimates the premium as the difference between the required return by the market and the risk-free rate. However, to compute the expected premium, historical premium prices are used, which lead to increased standard errors in the regressions (the authors report errors up to 10%). Combining those two, imprecision in forecasting cost of equity ensues. But before we delve further into this point, it is

[10] Vasicek, (1973) A Note on Using Cross-Sectional Information in Bayesian Estimation of Security Betas, *Journal of Finance*, Vol. 28, pp. 1233-1239.

important to examine some earlier studies which served as a stepping-stone on which the criticism over the CAPM was built upon.

Some early studies, such as those of Blume (1971) and Vasicek (1973), focused on the statistical properties of the Beta, in a more critical spirit. Specifically, Blume (1971) was a pioneer in examining how the Beta, as a standard of the risk associated with a firm, develops over time. The author argues that the ability of Beta to explain risk is related to the ability of the underlying model to accurately predict returns and rationalizes this through two approaches. The portfolio approach suggests that although risk is viewed through the prism of a complete portfolio rather than each stock's individually in the literature, that is not true in real world applications. To prove this point, he indicates earlier studies in which the variance of the returns is related to the variance of the market times the Beta of the individual stocks, plus an additional term of the residuals' variance. As the market variance is similar for all the investors, the only risk measure that matters, is the Beta of the stocks. The equilibrium approach is based upon the notion that the risk of each individual security can be expressed as a function of the market premium and the risk-free rate. The author, however, suggests that between the two theories only the first one provides some intuitive support for the Beta as an appropriate risk measure. The estimation of Beta proposed by Blume is the following (as presented in Lally, 1998):

$$\beta_j^B = \hat{a} + \hat{b}\hat{\beta}_J \tag{2.5}$$

Where $\hat{a}$ and $\hat{b}$ are the coefficients produced by regressing Betas estimated in one period against those estimated at a previous time-interval.

A noteworthy point of Blume's research is the summation of the three most important features of the contemporary literature (which will be under scrutiny as we will see in the following years and will spur the research to further focus on the underlying factors that affect a company's risk). Firstly, according to previous research, the relationship between risk and return is linear (as shown by the CAPM), secondly, he

explains that other factors do not contribute to returns but only in a rather small capacity (a mere 10%) and lastly all the unexplained residuals in the model do not have a specific structure between them. Blume's study also attests to the variability of the Beta over time. This is also extremely important and therefore highlighted several times throughout this thesis although a different methodological approach will be used. Blume suggests a correction based on historical rates of the Beta.

The proposed calculation is worth mentioning, as it is currently used by practitioners and academics (for example it is used by Bloomberg) to estimate system Betas, and it is based upon the idea that Beta coefficients vary over time. This in itself suggests that to get a better estimate about the future Betas, one has to simply regress the beta values of the current period with the ones from the previous. This adjustment, using historical Betas, improves the results for current Betas. This idea is extremely important, considering that one of the major criticisms over the original Beta is its dependency on the period we examine, with considerably larger periods pointing to Betas closer to the mean.

Vasicek (1973), also attempted to build upon the previous theory on the CAPM's Beta and expand upon it. Specifically, he examined this measure, in the context of Bayesian Decision theory, which employs and incorporates information through all the stages of the Beta calculation. The adjustment proposed, as explained in Lally (1998), is as follows:

$$\beta_j^{\nu} = \overline{\hat{\beta}}\left(1 - x_j\right) + x_j\widehat{\beta_j} \tag{2.6}$$

Where:

$$x_j = \frac{Var\left(\widehat{\beta}\right) - \overline{s^2}(\widehat{\beta_\iota})}{Var\left(\widehat{\beta}\right) - \overline{s^2}(\widehat{\beta_\iota}) + s^2(\widehat{\beta_j})} \tag{2.7}$$

This estimation connects the estimated Beta with the Beta of the peer group, as the variance of each observation is adjusted by the variance of the sample (for example the industry the company operates in). The author argues that although the calculation of the Beta is rooted in the principles of linear regression

analysis, the fact that the sample Beta is used to extrapolate on the Betas of the stocks in a portfolio has a major fallacy. This kind of estimation assumes that the true value of the Beta is known, which in reality describes a reversed situation, meaning that instead of the true value, an estimated value for the sample is known and inference for the calculations of new Betas is based upon this, which might lead to severe over- or under-estimations of the actual Beta of a stock, as the estimation does not include all available information. To counter this problem, the author suggests a correction, which is based upon the variance of the originally estimated stock Beta, and he explains through mathematical proof, that the new measure of Beta incorporates all available information. These results along with the ones provided by Blume, were supported by the study of Lally (1998), who, among others underlined the importance of examining Betas by industry classification.

Blume's research has been the basis for several other papers that criticized the CAPM, as the literature shifted towards other determinants that might affect the value of a firm's stock and the returns associated with it. Studies, such as the one made by Banz (1981), focused on how size might affect stock returns, and how CAPM is ill-specified as it does not account for a possible size-premium. Akin to these results are those of Fletcher (1997), who argues against the usefulness of the Beta, as it is more prone to be affected by downward trends in the market, but also concludes about the non-existence of a size effect in the UK stocks.

Other forms of criticism focused on the input of the CAPM. For example, Booth (1999) states that a primary problem of the CAPM is the fact, that it includes only one period for the investment, while in most cases cash-flows from an investment can be realized over multiple periods. The investment horizon is what determines the selection of the risk-free rate, when calculating the required rate of return on a stock. The risk-free rate is selected by the period over which the investment will be generating cash flows. However, the long-term Treasury bond yield is selected, when determining the cost of capital, as it is the one that most commonly matches the cash flows of investments, which is also a point of controversy between academics (who prefer the 3-month bills) and practitioners (who seem to favor longer periods). Similar

criticism is also made by Pettit (1999), who views the cost of equity derived from the CAPM as deficient. An interesting point that this author raises, is the case he makes against the use of longer period government bonds (he mentions the 30-year one). His reasoning is that in longer horizons stocks and bonds co-move.

Fama and French (1992), also focused on determining whether Betas are an important factor on determining the returns of a stock, with their results however being inconclusive, mainly due to the noisy nature of their data. Similarly, inconclusive results are produced by the study of Chan and Lakonishok (1993), who argue however that regardless of their own results the problems of the Beta, namely the time frame chosen and data availability, overshadow its ability to accurately predict stock returns.

Although the CAPM has been criticized heavily, it has also seen considerable support, mainly as researchers began to experiment with different time frames. Fernandez (2006), is one of those proponents, as the results from the wavelet analysis[11] she uses, allows her to find evidence in favor of the CAPM at the 4-16 days horizon, as the predictions made by the model tend to be closer to the real outcome. Similarly an earlier paper by Homaifar and Graddy (1990), provides empirical support for the Beta of the CAPM, as they say it is less biased than others estimated through other contemporary models.

Estrada and Vargas (2012) argue that Beta has been the epicenter of major controversy in financial literature for the past few decades and explore the effect that negative circumstances have on investments and how those are impacted on the Beta. They find that the Beta moves in the direction of major events (increases for negative events). Further support is provided by the contemporary study of Da, Guo, and Jagannathan, (2012), who suggest that the usage of CAPM might still be one of the optimal ways for investors to estimate the cost of equity for an investment, as it is capable of accurately predicting the cross-sectional variation of the option adjusted risk premium through the use of an option adjusted Beta.

Finally, a prominent paper that focused on the market Betas as a means to incorporate information and reflect any information changes, is that of Cosemans, Frehen, Schotman, and Bauer (2016). The authors

[11] Wavelet Analysis is the methodology by which the original time-series data are decomposed to time-scaled variables, where the frequency of each one of them contains significant information on the original dataset (Gustafson, 2014; Ramsey, 1999)

argue that the study of asset pricing through the use of portfolios based on specific characteristics (as is the norm in current literature), is flawed. The idea is that the underlying factor structure in those portfolios is what determines the explanatory power of any model employed to explain the risk premiums, as information on other factors than the prevalent ones is lost. To counter this problem, a new model is proposed to estimate company Betas, or hybrid Betas as the authors call them, which reflect all information associated with the firm's fundamentals, which is created with a rolling square error shrinkage technique based on the proposal of earlier studies such as that of Vasicek (1973) and Chan, Karolyi and Stulz (1992). The proposed model is as follows:

$$\beta^{*}_{it|t-1} = \delta_{0i} + \delta_{1i}X_{t-1} + \delta'_{2}Z_{it-1} + \delta'_{3}Z_{it-1}X_{t-1} \quad (2.8)$$

Where X_{T-1} reflects the business cycle variables and the Z_{it-1} the lagged firm characteristics. The results indicate an even higher error reduction percentage, ranging from 15% to 25% depending on the horizon of the sample, as well as highlighting the inability of the Fama-French portfolios to properly incorporate estimates of the Betas for individual stocks within the portfolios, as information on those stocks is lost in the process. The authors conclude that their proposed hybrid Beta results contradict the idea that Beta estimations are of low value to investors and explain why they are so popular in practice.

Regardless of whether one agrees or not on the validity of the CAPM, as a model suitable for predicting stock returns, one can only admire the resilience of it throughout time, especially since it has been proven in several occasions that better estimates than the Beta exist. The continuous acceptance (and use) seen by not only academics but also practitioners, can be attributed primarily to the simplicity in its implementation, and arguably (based on the studies mentioned above) to its ability to roughly predict expected returns. This does not mean that it is without limitations, as it can be seen from the various studies noted here that it does not account for a large portion of the risk that should be reflected in the expected returns. With this in mind we will proceed to explore the alternative models that have been proposed in its place.

2.4.1.4 Debt-related Betas

Debt is a form of financing that can be mainly classified to bonds loans and overdrafts (Ogier, Rugman and Spicer, 2004). The structure of this means of financing, as well as the fact that the payments towards the repayment of the debt, come directly from corporate income and in the case of bankruptcy debt providers are compensated before equity holders, reduces the risk associated with the investment significantly. Even the way diversification works for a bond portfolio is different than that of equity. For that reason, literature on the cost of debt is relatively sparse, but nonetheless significant. The most important issue that arises from studying debt, is how to properly balance the total cost of capital between debt and equity, and what an appropriate level of debt would be in order to finance a company without putting excessive stress on it. Besides the Multifactor Models and the Arbitrage Pricing Theory, another family of models was created, driven by the ideas of a seminal theory created by Modigliani and Miller (1958).

The first study we had to review, as we set to explore this topic further, is that of Hamada (1972) and explain why it acted as the cornerstone for most of the papers that followed in this research area. This study begins by explaining how the CAPM is defined on the basis of merging the ideas of borrowing and lending at a fixed rate (the risk – free rate) and the market portfolio with the ideas of Modigliani and Miller, in the sense that any additional unit of leverage raised by companies or investors, while simultaneously holding the equity at a specific level, will make the risk associated with them surge, and subsequently entrain the non-diversifiable risk towards the same direction[12]. He then proceeds to first explain mathematically the relationship between Beta and leverage, followed by testing the Modigliani – Miller Theorem, through the use of specific case studies (he uses the cases of the electric utilities and the railroad industry).

[12] The author uses the Debt-to-Equity ratio as a measure of leverage.

Hamada's findings support the theorem, and he explains that up to 24% of systematic risk can be attributed to the debt to equity mix of a company. Hamada's paper is important in the private companies setting as well. It can be used to obtain an unlevered Beta for a private company which can then be levered iteratively until equilibrium is reached on market value constituents. This study is extremely important, as it became the basis for the creation of the accounting Betas' theoretical stream, which we will analyze further in a following section of the review.

The literature has pointed towards the inverse relationship between returns and leverage in several occasions. For instance, Muradoğlu and Sivaprasad (2012), not only confirm this notion, but they also find that lower leverage is associated with higher returns in the long run. The unlevered Betas seem to perform better in the study of private companies' cost of equity. Sarmiento-Sabogal and Sadeghi (2014), come to this conclusion, along with the fact that the Modigliani – Miller approach[13], under which the absolute value of debt remains constant through time and the rate to discount tax shields is the cost of debt, generates more robust results, however this approach also seems to overestimate systematic risk.

2.4.1.5 CAPM Variations

The CAPM is important, because it is a method of estimating the undiversifiable part of the risk, the systematic element. Early studies though suggested that to truly achieve diversification, an investor must have portfolios that hold stocks from a variety of markets, because in this way they are capable of counteracting country risk. This notion stems from the assumption that in order for an asset's cash flows to be similarly valued regardless of the country that they are generated in; international markets should be fully integrated. Full market integration allows investors to be fully diversified. The research paper of Stulz (1981), was based on the idea that there have been no reliable models to test on whether markets are truly

[13] The Modigliani- Miller approach is based on the seminal theory created by Modigliani and Miller, (1958), which suggested that in a world with no frictions, or other costs (such as taxes) and within an efficient market the value of a firm is not linked to its capital structure

partitioned or not, because most of them relied on the assumption that asset pricing is tied to the exchange rates and consumption capabilities of investors, as those are determined by their country of origin. This controversy led him to develop an international version of the CAPM, that operates under the assumption of full market integration with differences in consumption opportunities. Instead of deviating from the previous literature, the findings suggest that there is a proportional relation between home country returns and changes in consumption rates, however he adds that money supply is also capable of explaining cross-country variation of returns. These results are supported also by the research paper by Chan et al. (1992), who find that its results remain significant when different markets are examined (they also conclude that international markets are co-integrated as we will see at a later section).

The international CAPM has been met with considerable support over the years. Several studies report the ICAPM's ability to effectively measure returns globally. For example, Fernandez (2005) explores the explanatory power of the international version of the CAPM, which incorporates both market and exchange-rate risk, as well as, proposing a model that uses time-scale VAR and marginal VAR analysis. His results support the use of the ICAPM but also reports that not only the stock market for which the methodology is applied matters, but also the investment horizon. Support for the ICAPM is also given by the paper of Kurach (2013), who finds evidence that support the idea that local stock market volatility is a by-product of the global one, as markets are co-integrated, and as such Betas are a good predictor of future returns, while they can provide a framework for potential international diversification opportunities. These findings are important, since they signal that investors value idiosyncratic risk similarly regardless of the market, they invest in.

Another alternative model to the CAPM was the consumption version. While the classic CAPM uses the covariance of the stock together with the market itself, the consumption version (or conditional version) of the CAPM uses the covariance of the stock's return together with the per capita consumption. The CCAPM is, according to its supporters, a better measure of portfolio performance as it incorporates more factors than the classical version of the model, such as the overall wealth of the investors. Chen (2003), provides

empirical support for this version, as he compares how both of them perform, and finds that the original CAPM provides investors with more accurate results, at explaining the stock's performance over time. These findings are also consistent with those of Santos and Veronesi (2006), who attempt to expand on this version by including an income to consumption ratio, in an attempt to highlight the importance of labor income and human capital. They provide support for the original CAPM however, they suggest that the results are vastly improved by the inclusion of the aforementioned ratio.

The CAPM has also been used to examine how risk in asset pricing is determined from the supply side of investments. To exemplify that idea, we refer to the study of Lee, Tsai, and Lee (2015), who examine a variation of the capital asset pricing model, the dynamic generalization of the static CAPM, that views investments and their risk through the eyes of the supply side. As this is a rather unique occurrence (although it has been mentioned again in the literature see for example Cox, Ingersoll, and Ross, 1985), we will consider this in more detail. They make use of two different types of tests, one based on price per share and another based on dividend per share, in order to find the degree that the supply effect determined the asset prices, with data found on the S&P and Dow Jones Indices. The supply effect seems to have major impact in US stock markets, when companies are examined both individually and as part of portfolios.

Although the previous versions of the CAPM seem like straightforward ideas, as to how to improve the Betas reported through the original model, there were other more exotic versions, that have been developed more recently. One prime example is the study of Terceño, Barberà-Mariné, Vigier, and Laumann (2014) who use a novel methodology, widely known as fuzzy regressions, to improve the Beta coefficients used, in an attempt to estimate by including the effect any shocks or management decisions, may have on them. As it is stated, the CAPM, the most renowned model used in Finance to capture the systematic risk associated with a stock, views the relationship between stock and returns as linear. However, in order to estimate the Beta, historical prices must be used, which implies that all stocks are affected majorly by their market. They also argue that the other assumptions, which reduce the efficiency of the model, have also to be made. In the end though the Betas remain stable regardless of the probabilistic or fuzzy nature of the

approach, they tend to be more stable in longer horizons and when the overall industry sectors are used and not the sub-sectors.

These variations of the CAPM, although they initially seem to provide support for it, they underline one of its major fallacies. It primarily focuses on one factor only and leaves out others, that becomes evident by the plethora and variety of the other iterations of the original model, are important in the explanation of the expected returns. This is a recurring theme with the CAPM and as we will see pushed researchers towards other means of estimating the returns.

2.4.2 Macroeconomic Factors

Although we have already touched on the topic of macroeconomic factors in various sections of the review (for example we discussed how macroeconomic factors affect the equity risk premia), we believe that this particular stream of the literature deserves a separate section, as the overall performance of the economy can be linked to the performance of the firm, as it affects both the supply and the demand sides of the products. The overall status of the economy is so important that appraisers begin their valuation reports usually by discussing topics such as inflation, short and long term interest rates and the overall output of the country that the company under examination, operates in (National Association of Certified Valuators and Analysts, 2012). Macroeconomic theories attempt to relate expected returns with the macroeconomic variables mentioned previously (Cooper and Priestley, 2009; Neely et al., 2014; Strong, 2003). Regardless of whether a market involves transaction costs or not, consumers share risk, as the underlying factors are common to all of them (Cochrane, 2011). This realization led researchers in the pursuit of these variables that can affect multiple consumers and subsequently the businesses as well. Two distinct categories in these types of models is the investment-based and the general equilibrium ones, with the former focusing on the relationship between expected returns and investment decisions and the latter seeking to examine the effect of new technologies and fundamental shocks on the economy.

As the literature is replete with studies examining the relationship of abnormal returns with a number of macroeconomic determinants, we begin this review with the early study by Chen (1991), who focuses on the factors that affect future consumption and investment opportunities and are associated to the capital markets. This is in the sense that capital providers consider them in order to calculate their expected returns on their investment. Production growth, short-term interest rates, long and short-term bond yield spreads and dividend yields, seem to be closely related to asset prices, stock returns and premia. His findings suggest that market excess returns are negatively associated to current, but positively associated to future, economic growth, and offer further support to the argument that returns can be predicted by the same macroeconomic factors that affect the overall economy. Similar results are reported by Bali, Cakici and Chabi-Yo (2015), who find that the risk investors are willing to take is related to macroeconomic factors, such as the GDP, the unemployment rate and the overall risk of the equity market itself.

All of the above allow consumers to shape their beliefs about the prospects of the economy and can be indicative of whether the economy is in a positive or a negative point in its cycle. These ideas are presented in research such as that by Fisher and Statman (2003), who explore the relation of consumer confidence on stock returns, as well as, investor sentiment. The authors use the University of Michigan's Consumer Confidence Index (CCI) in conjunction with the Conference Board Consumer Confidence Index and find that high consumer confidence is usually related with low stock yields, however there is no strong connection between consumer confidence and the S&P 500 returns. The relationship however is inverted when seen from the perspective of the movements of stock markets. Upward movements in the stock returns are followed by an increase in consumer confidence, while the opposite does not hold. The same pattern is also revealed for the relationship of investor sentiment and stock yields.

The same topic has been investigated several times throughout the literature. For instance a later study, presented by Schmeling (2009) who explores the effect of consumer confidence, which he uses as a means to approximate investor sentiment, on stock returns in a number of industrialized countries, as was done in the study of Fisher and Statman mentioned previously. The approach that is used in this paper is important

because Schmeling focuses on countries with lower levels of institutional growth and those that are more prone to display a herd-like behavior. He then contrasts those against the results from previous literature, concerning the US, and concludes that, investor sentiment is an important factor on stock returns, across the globe. In addition, he notes, that there is a stronger impact on returns, in countries with less developed markets and those that are more susceptible to herd-like behavior.

GDP growth is a well-established determinant of stock returns, as it is tied with the prospective growth in business cycle (increased demand and subsequently higher production). Rapach, Wohar, and Rangvid (2005) and later Rangvid (2006), focused primarily on this topic. The former's findings suggest that GDP growth together with interest rates are the most suitable in determining prospective stock returns, while the latter showed that a large portion of the variation on stock returns, both normal and excess, is captured by the price to GDP ratio, while the more common ratios used, namely the price to earnings and the price – dividends, are not as related to returns as was suggested by previous research. These results are further strengthened when a longer horizon set of regressions are used as those tend to diminish the noise in the data, through the accumulation of returns.

Although GDP growth is generally accepted in the literature, there are voices that point to the contrary. Cooper and Priestley (2009) state that it is highly important to be able to accurately predict the stock and bond returns, within a trade cycle frame, as this will increase the understanding of time-varying risk premiums. However, the typical macroeconomic variables used, such as the GDP growth, or the sentiment variables, have been proven to be inadequate in accurately predicting returns. For that reason, they propose a new measure, in the form of the output gap, which has several prevalent characteristics as opposed to the other financial and macroeconomic characteristics. For example, the fact that it does not incorporate the level of asset prices and the fact that it is linked to production-related data as opposed to the classical macroeconomic variables, which are based on consumption data. Moreover, as opposed to prior literature findings, the results indicate that short term returns are predictable, with the use of output gap, and they are robust regardless of the time-period chosen.

As we have discussed, country premiums have been suggested by researchers as an addition to the international version of the CAPM. However, these premia are reflective of several other underlying factors that characterize the country that a company operates in. One such determinant is the political risk. Quantifying it however has been difficult, and the usual practice in the literature is to use the spreads in the sovereign bonds. This practice is however erroneous, since it results in higher cost of equity for the firms as Bekaert, Harvey, Lundblad, and Siegel (2016) explain. They suggest a new measure of political risk, obtained by the spreads, in specific countries (that acts as a benchmark). The authors argue that the standard practice of using the countries' specific spreads (difference between that country's bond yield and the equivalent US treasury yield) encapsulates the fallacy of accounting for the systematic portion of the risk twice (as the risk-free rate is inflated by counting the political risk an additional time). Political risk is particularly important in the premises of this thesis, as it is a major determinant of the discount factor, in FDI decisions, since it is negatively associated to cash flows, when investing overseas.

Other studies shift their attention towards market trend, or momentum. Gompers and Lerner (2000) link higher expected returns (and more favorable valuations for the stock that is being valued), with the increase to the inflow of capital in the stock markets. This inflow is spurred by venture capital funds who attempt to take advantage of the upward trend in the stock market, and thereby creating further increases. Similarly Demirer and Jategaonkar (2013), find that the risk associated with return distribution is highly associated to positive premiums in upward trending periods for the markets, under examination.

Momentum is also fueled by the links between markets. Fama and French (2012) attest to that notion by examining international stock returns through the prism of value premiums and momentum, in a number of regions around the world. Their findings suggest a significant relationship between returns and momentum in almost all developed markets (apart from Japan), as well as to the fact that developed markets seem to be highly linked with each other. The link, or co-movement, of developed (and under-developed) stock markets is often responsible for shaping consumer confidence, which as we have discussed is one of the major determinants in explaining expected returns. Finally, Barroso and Santa-Clara (2015), point out that

momentum is incorporated in investment strategies, and it has a greater effect during the negative shocks that affect the markets, which explains why it has been viewed as a dead anomaly throughout the last decade, that included numerous shocks and events.

As one can easily notice, macroeconomic factors have been established to affect expected returns. The interesting part is that regardless of the methodology used (whether it is the CAPM or any multifactor model), they are always considered as they have been proved to affect value. This is the reason why we will include the most prominent ones, in this study, as it will allow us to capture that aspect of the risk.

2.4.2.2 Arbitrage Pricing Theory and Multifactor Models

CAPM is the most commonly used model, when calculating the cost of equity. However, as we have already seen, it has also been a field of controversy for many academics, who dispute its applicability in the real world, since it fails to fully explain historical stock returns and due to its simplicity, it seems to not take into consideration all the possible factors that may affect the cost of equity. For that reason, researchers have tried to improve CAPM or develop additional models that better explain stock returns. The most renowned in the latter category are the Arbitrage Pricing Model developed by Ross (1976) and the various multifactor models, proposed by authors such as Fama and French, although it is a fact, that the CAPM is the model that sees the highest acceptance by practitioners, it is judicious to take a closer look at the literature of the alternate models proposed, to consider whether any further insights can be drawn.

2.4.2.3 Arbitrage Pricing Theory

The Arbitrage Pricing Model holds a special place in the alternative models proposed to the CAPM. It was originally developed by Ross (1976), as an alternative to the CAPM, as he suggested that the linear relationship that is implied by the mean-variance model, can hardly justify the normality in returns. This

model holds in situations where there is no equilibrium (which is the principle that the CAPM is built upon), however there are weak points in it, with one of them being that with the increase in the number of assets in the portfolio the wealth will also increase, however in practice as the assets increase so does the total risk of the portfolio, thus leaving the total return close to zero (the increase in total variance will decrease the total returns eventually). This seems counter-intuitive in the beginning however it becomes clearer if we review how APT is constructed, as the inclusion of an extra risk factor will cancel out the variance of a previous factor, eventually bringing the total returns to zero (or the risk-free levels of return) given a large enough number of factors:

$$E(Ri) = Rf + (\beta 1K1) + (\beta 2K2) + \ldots + (\beta nKn) \quad (2.9)$$

Where:

$E(R_i)$ = Expected rate of return on the subject security
R_f = Rate of return on the risk-free security
β_1, β_n = Sensitivity of the security to each risk factor relative to the market average sensitivity to that factor
K_1, K_n = Risk premium associated with factor K for the average asset in the market

Ross's paper was one of the first to argue against the Betas and the CAPM, with others following fairly quickly. The study of Chen (1983), compared the APT and the CAPM, and suggested that Ross's arbitrage pricing theory model is the most efficient one at explaining the cross-sectional variation of asset returns. The main hypothesis developed in the model, is that every asset is linearly related to a pool of determinants and to its own idiosyncratic volatility, an assumption supported by his results that pointed towards better predicted returns with the inclusion of common macroeconomic determinants.

Further support for the APT was provided by other studies that focused on businesses' operation cycles and the factors that affect them as a means to interpret expected returns. For example in the study of Kroll and Caples (1987), they explain that modern firms are often viewed as an assortment of businesses, in the sense that each part of a corporation performs a specific and distinctive role. There was an effort by various corporations, to better understand how to increase the productivity of each part, by examining a number of

individual factors that affect the cycle of operations or the industry, the businesses operate in, as a whole, and recognize synergies as a means to create value.

The main problem with these multidimensional models though, was that they oversaw key financial and economic factors, and the approach to value was mostly done through qualitative means and did not account for the cost of equity and debt faced by the firms. This supports the role of the Arbitrage Pricing Model, since it connects the intuitive determinants of value together with the key performance indicators of a firm, and it allows for a more accurate estimate of the corporations' fair value, in an efficient market (Goodman and McLelland, 2015; Wei, 1988).

2.4.2.4 Multifactor Models

As we have seen several studies have suggested that the CAPM, fails to take into consideration significant factors that are critical in explaining expected returns. For that reason, multifactor models were created, that focused on connecting returns with specific factors, with much of the empirical work being based on studies conducted by Fama and French (Fama and French, 1993, 2012, 2015). In general, these models focus on macroeconomic and fundamental factors, with prime examples being GDP growth, inflation and consumer confidence (all of which will be applied in this research) together with assets, as a proxy of size, and earnings. We will be analyzing the principles of factor models in the methodology section, so it suffices for now to say, that these models expand on the CAPM, which was a single risk factor model, as a method of complementing it, to account for return over-time variation.

The review of the literature on this area must start with the early study of Fama (1981) who examines the idea that inflation and stock returns, have an inverse relationship, contrary to the popular belief that stocks, are income created by real assets and thus should be used as a safe haven against inflationary trends. He attributes this relation to the effect inflation has upon the real economic activity, a hypothesis also backed by his data, with which he finds that abnormalities in stock returns, can be smoothened and explained by

real variables, namely the quantity of investments available to firms, with the rates of return exceeding the cost of equity for the firm, and the inclusion of expected and unexpected inflation. A few years later, another study by Fama and French (1988), suggested that stock prices are affected by a mean-reverting process, which in turn results in negative autocorrelation in returns. Autocorrelation is related to the horizon of the data, with long horizons being associated to higher market efficiency. This also indicates that there is a relationship between price variation and return variances, as well as with the size of the firms under review.

The results from the previous study were cemented by a later paper of the same authors and created the most renowned form of this multifactor model, the three-factor model. Specifically, Fama and French, (1992), introduced two easily obtained but highly important variables, company size and book-to-market equity, to explain the variation in stock returns. They use those variables in conjunction with market Beta, leverage and earnings-price ratios, to improve the CAPM. These results suggest that the size factor inclusion allows us to predict average stock returns more accurately. In the same paper Fama and French argue that the conclusions extracted by their three-factor model, can be applied to other asset classes beyond stocks and more specifically, bonds and this is a clear indication that stocks and bonds are linked through their risk premia. The implications of their finding, which is consistent with others throughout the literature (Cochrane, 1991, 2011), are of particular importance, within the premises of this thesis, as it can be argued that the discount rate (or risk premium, or expected return), is the link that allows researchers to extrapolate their conclusions and thereby create a unified framework for asset pricing.

The same idea can be seen in several other prominent studies that have focused on it in earlier years. For example the one conducted by Freeman (1987) examined the connection between stock returns and accounting earnings for small and large firms, in terms of when they happen and how significant they are. His findings suggest that large firm equity prices are faster to incorporate reported earnings than those of smaller firms, with the twist being that the abnormality in returns is reversely analogous to the size of the firms. Interestingly, this argument is also in accordance with the theory that suggests that information production for the purpose of finding mispriced securities can be modelled as a function of the firm's size.

The two factors identified above, namely size and book to market equity, have seen great support throughout the years. For example Clare, Priestley, and Thomas (1998), report that these factors are the main drivers of the stock return variation in the UK (however to be complete, we should mention that they also find some support for the CAPM, in a specific setting that revolves around the inclusion of idiosyncratic returns). Another study that examined the UK stock returns in conjunction with the Fama-French three factor model was that of Gregory and Michou (2009), which is interesting as it opposes all the various methodologies CAPM, three factor model and the four factor model proposed by Carhart (1997), and finds that all models outperform the CAPM, and the results being similar for the other two models if the investment horizon under examination is short.

Multiple methodologies used in asset pricing are examined in several papers and most of them provide support for the size and book to market factors. For example, Gospodinov, Kan, and Robotti (2014), concluded that the impact of macroeconomic factors was minimal. Similarly, Maio and Santa-Clara (2012), also examine several models and conclude that the best results in the prediction of expected returns can be achieved through a combination of the ICAPM and the factors identified in the three-factor model. This is a significant finding as this is the first research paper that combines the results from the two methodologies. Another study, with a similar approach of testing the various methodologies, is that of Bartholdy and Peare (2003), followed by Bartholdy and Peare (2005), with results that indicate the inefficiency of the original version of the CAPM in predicting expected returns, as opposed to the multifactor models (specifically, they use the Fama-French three factor model), something they prove through using different time periods of data, which they claim produces slightly better results, in a 5-year span of data of stock returns. These last two research studies add to the preexisting literature, in the sense that most of the theoretical research completed in this area tends towards the higher return estimation capabilities of the Fama-French multifactor models as opposed to CAPM-based ones.

Another factor that has gained significant acceptance and should be mentioned at this point is momentum, which indicates the “traction” that a stock’s price gains and the trend that it follows. Momentum has gained

considerable acceptance amongst traders as it is considered an indicator that can be used in technical analysis. The original study of Carhart (1997), suggested that price momentum, measured by prior period returns, should be included together with the other three factors identified by the Fama-French studies. These results are further strengthened by those of similar (and more recent studies), such as the ones of Kothari et al. (2006) and Maio (2015). It was also examined by Fama and French (2012), who identified it as one of the factors that should be included in the analysis of the expected returns.

The discussion about the additional factors is ongoing, as is evident by research papers such as the one of Fama and French (2015), who incorporate profitability as a determinant yet find that additional factors beyond the four original ones get absorbed by the rest. There is however, the pertinent question of the number of optimal factors which has also been explored, even in the early years that this theoretical stream began evolving. Connor and Korajczyk (1993), try to determine the adequate number of factors needed, not in a strict sense but rather by approximation, based upon the idea that factor analysis is not deterministic in the sense that the actual number of factors that should be included is unknown. They develop a model, under which the appropriate number of factors, do not significantly decrease the mean of the returns and find that the optimal structure of factors for this model, includes one to six factors.

The factor models have been perhaps the most recent and most generally accepted method of asset pricing, especially by academics (practitioners favor the use of CAPM as Brotherson et al. (2013) suggest). As Cochrane (2011) argues they are very useful in the sense that the theoretical framework of asset pricing can now be focused in explaining the premia in the High minus Low (HML) factor rather than the returns of the various assets. The centerpiece of this theory is covariance, and the subsequent cointegration it suggests. However, as it became evident (by the inclusion of momentum as a factor for example), that there might be other determinants that need to be explored, besides the ones indicated by the three-factor model. This is a problem with factor analysis in general, as the factors are in essence unobserved variables. Regardless, the transition from the single-factor CAPM to the multifactor Fama-French model, was a significant improvement, as it came with the realization that it is impossible to explain the expected returns without

admitting that they can be affected by a multitude of underlying variables. This idea is one of the cornerstones of this thesis, as we attempt to examine the P/E ratio, and the determinants that affect them.

2.5 Unsystematic Risk

Unsystematic, or diversifiable risk is the uncertainty associated with investing in a specific company or industry and can be diversified away by investor's spreading their available wealth over different companies (Chittenden, Poutziouris, and Michaelas, 1999; Dailami and Hauswald, 2007; Ľ. Pástor and Veronesi, 2006). While this might work for investors with portfolios that include public companies, it is not so easy to be done in any investment that cannot be traded in an active market and which constitutes the major part of an investor's wealth. To understand the severity of the issue, we can consider Ljungqvist and Richardson (2003), who explain that private equity suffers from high illiquidity. This is due to the lack of availability of an active secondary market, as well as the reduced control investors have over their investments, which require them to "bind" a significant part of their available funds over a long time-period.

The problems presented to investors of private enterprises are similar. Livingston (2014), for example, opens her study on the discount rates for private businesses, with the remark that private firms are "branded" by illiquidity and opacity. For that reason they also have to pay higher return premiums, as a compensation to their investors. Butler and Pinkerton (2006), explain that this premium originates from the inability of the investors in a closely-held firm to estimate the correlation of it with other investment options, they might have. Based on the theories that have been developed thus far, this argument seems to be correct, as the idea of reducing investment risk is indeed tied to the idea of diversification, which in this case is not applicable. Akin to this idea, is the study of Graham and Harvey (2001), who suggest that a large number of the firms they examined, preferred using firm risk rather than project specific risk when considering whether the project will produce the expected cash flows, when evaluating an investment. Contrary to other

studies, they also indicate that transaction costs, free cash flows and asymmetric information, are of less concern to corporate executives than the risk associated with the projects they undertake.

As we will see private companies are subject to higher return premiums. Abudy et al. (2016), expanded upon a previous paper by Polk, Thompson, and Vuolteenaho (2006), which suggested amongst other points that there is no linear relationship between company specific risk as expressed by the Beta and the stock's returns. Abudy et al. (2016) focus on that premium imposed on transactions involving private companies and argue those are dictated by the company's leverage, the taxes associated with the transaction but most importantly the level of (non-) diversification of the company's owners. The latter is especially, important and we will expand on it, as it is one of the defining characteristics of private business valuation. In addition, we will also visit a set of variables that affect the unsystematic portion of the risk, which can be broadly categorized as industry and company specific.

The industry a company is part of, is highly important, as most practitioners adjust for it with a risk premium, when they perform a valuation, in order to encapsulate risks specific to this industry that affect only companies that are part of it. Barad and Mcdowell (2002) argue that Ibbotson attempted to deal with this issue, by developing and publishing his own risk premia, through the use of data from all the participating companies in an industry. Furthermore, they suggest, that the full information approach that is employed, estimates the individual risk each company brings to the industry. Through the use of this metric, appraisers can accurately estimate the premia that needs to be assigned to each company being valued. Although this is partially reflected in the CAPM (as only a part of industry risk can be diversified away), this is not the case for the build-up method, which we analyzed previously and is used primarily in the valuation of private enterprises.

2.5.1 Industry

2.5.1.1 Legislative and Tax Value Drivers

We will begin the analysis of the factors that are industry-wide, with the legislative and the tax-related ones. The reasoning behind this is that those variables, as we will see are associated with higher stock volatility (and in some occasions higher trading volume), with abnormal returns and of course, as we explained in the introductory section with value.

We will begin the review on the impact of taxation with the study of Brennan and Schwartz (1978) who examine the effect corporate income taxes have on the relationship between the optimal capital structure and company valuation This effect leads to the conclusion that debt is the only element that the perfect optimal capital structure should consist of. However, this contradicts the most basic principle of corporate finance, shareholder's wealth maximization. Earlier studies attributed this fact to various reasons, ranging from income tax, in the sense that those might reduce the cost of retaining earnings, to a mere attempt to join theory together with practice, or perhaps the simple reason of managers just wanting to secure their own position within their companies. The consensus is, similarly to the results of the aforementioned research paper, that the maturity of debt is what affects an optimal leverage ratio, and more specifically short-term debt can be issued continuously, in order to reap the benefits of tax savings as well as avoid bankruptcy.

Legislative and tax-related factors have also been associated with abnormal stock returns as shown in the studies conducted by Poterba and Weisbenner (2001) and Ivkovic, Poterba, and Weisbenner (2005), as investors try to take advantage of beneficial laws that allow them to realize gains through taxation. Specifically, Poterba and Weisbenner (2001) explain that research to this point has focused mainly on three ideas, as possible interpretations on the abnormal increase in stock returns by the end of the year. Specifically, cash flow themes, end of the year reporting requirements and income tax reasons seem to be the prevailing factors put forth by the literature. Encouraged by the notion that differences in capital gains

taxation affect individual investors more than financial institutions, the authors propose a test that they say is able to discriminate between those factors and determine whether window-dressing or tax-loss selling is accountable for unusual year-end returns. Moreover, their test is used under different tax systems, and the results point to a strong relationship between tax laws and return patterns, as well as, tax-loss selling as a defining determinant in the returns.

Tax-related investment incentives are created due to relative legislation, as Ivkovic, Poterba, and Weisbenner (2005) suggest, as investors may be able to realize some after-tax gains, by mitigating the damage from the underperforming stocks. Their results indicate that, there is a lock-up period for capital gains, and there is a trend of tax-loss selling in December (or 31st of March for the UK corporations and 5th of April for individuals), that is stronger for losers than winners, while this effect is inverted for the rest of the year. Investment decisions during the last month of the year appear to be affected by the tax-loss selling effect, and stocks sold during this month are less likely to be rebought in the upcoming months. Ultimately, however, investors will realize higher after-tax returns if they use tax avoidance strategies, that help speed up realizing losses. On the topic of tax-related investments and strategies Faff, Hillier, and Wood (2001), who explore the relationship between taxation over dividends and low Beta portfolios, find evidence that taxation laws have a negative effect upon the Betas.

The popular belief in the literature is that capital gains taxation has an adverse effect on stock returns, in the sense that it is preferable for investors to realize losses and thus reduce their tax burden. Under this assumption, capital tax gains should be creating specific investment behavior patterns (which are over recent years a part of the behavioral theories, developed under the umbrella of behavioral finance). This notion is put under test by Dyl (1977), who examines year-end trading volumes of common stocks and how those might be related to capital taxes. The evidence supports the common belief that investors sell en-masse stocks with falling prices, so as to realize losses and subsequently face lower taxation.

Other forms of legislation have been proven to have an effect on stock returns. Leuz (2007) reviews two previous studies, from earlier in the same year, that have been conducted in regards to Sarbanes-Oxley Act

(SOX) and its effect on stock returns and the decisions of companies to go private, and argues that although there has been significant work done by predecessors, there are some critical issues that need to be discussed, especially on how the results of each study are interpreted, and whether those are related to the legislation itself or can be attributed to general market trends. For the first study Zhang (2007), the one that analyses the effect that SOX had on stock returns, Leuz, suggests that there was no control group for unaffected firms, while simultaneously the time frame under which the study was conducted, thrives with events that could have a negative impact upon the performance of the stocks. For the second study Engel, Hayes, and Wang (2007), the author, argues that it faces the same interpretation problem that the previous one had, since the study might overstate the fact that SOX imposed more costs for smaller firms that eventually decided to go public. He concludes with the need to not rush to conclusions, when interpreting the empirical evidence, particularly on the topic of the burden that a certain legislation might impose upon companies and their ability to raise capital.

Other studies, such as that of Dixon Wilcox, Chang, and Grover (2001) explain how legislation can drive the economic activity forward. The authors suggest that a recent legislation change in the US regulatory framework, regarding telecommunications, resulted in a wave of mergers and acquisitions, as well as, partnerships between companies, of various sizes, in that specific field. In particular they examined the effect of this specific piece of legislation on the M&A activity and the value of the firms involved, in conjunction with the synergies created for those firms and how those impacted their value, for which they find significant support. Similar results are reported by Hail (2013), who focuses on the convergence of the IFRS with the US GAAP, and how the trend of regulation strengthening affected the valuation process.

Legislation effects have a significant impact on all affected companies, which might be even more taxing in the case of private businesses and might even affect their decision of whether they will go public or they will stay private. These are the results shown in the research paper of Helwege and Packer (2009), who state that private US firms are considered as more prone to going public than companies in other countries, something that might be attributable to the minority interest protection offered by US legislation, or the

high liquidity and diversification gains. However, that is not the case, as the evidence suggests that private equity firms are buying out public companies and taking them private. Even when private companies, have easier access to cash (admittedly not often), they prefer to use this cash, not to actually go public, but to maintain a high leveraged position, in order for the management to be properly incentivized. Another issue regarding legislation is brought up by Eije and Wiegerinck (2010) who indicate that it is an established fact that private firms are mostly the targets of cross-border acquisitions. Things however become more complicated due to different legal frameworks, under which these acquisitions occur.

Other studies, such as that by DiGabriele (2008), explore the impact of the Sarbanes-Oxley Act has on private company valuation. He discovers, that the discount rate after SOX, increased significantly, while he also compares the effect of the legislation change on both private and public companies and concludes that private firms took a larger hit, due to the fact that they had to bear the significant changes in their due diligence processes and mainly because the non-compliance with the SOX provisions, would be a bad signal to potential acquirers.

Based upon this analysis, the importance of legislation is highlighted. As it seems favorable legislative acts might even spur M&A or going-public activities, and therefore directly affect value. For that reason, a group of legislative acts will be covered by the proposed model, as we want not only to examine the effect legislation has on the discount rate, but also how different legislative frameworks, impact the valuation process in different countries.

2.5.2 Company – Specific Factors

In this section we will be covering several factors that have been found to affect businesses, both private and public. We have already briefly mentioned some of them, throughout the literature review, however at this point we will consolidate them so as to make it easier to consider a more complete picture of the ideas that have been developed. This part of the literature is akin to this research project, as the aim with it was

to identify all the determinants of rate of return applied in valuation from the literature, and if possible include them in the model development, so as to allow the unique methodology to eliminate the less significant ones and thereby allow researchers to focus on those that have the most critical impact on the discount rate.

There has been an effort by some researchers to link company value to various firm-specific characteristics, ranging from the type of debt the company has, to other features such as the control an investor wants to have over the management of a company. For example, Barclay, Holderness, and Sheehan (2007) state that investors engage in private placements either in an attempt to better control a company's management, or just to emit a positive signal to the markets regarding the fair value of the firm. However, another theory, that of managerial entrenchment, suggests that these kinds of strategic moves are done in order to ensure that the investors in a company will not try to alter the status quo within the firm. However, according to the authors of this study, the samples used in previous studies were not sufficient to extract accurate conclusions on which of the aforementioned hypotheses, explains better managerial entrenchment. They find that the reaction of the markets to the announcements is linked with the type of the private placement's buyer. Active investors are associated with positive signs on the long run performance of the company's stock. Perhaps their most interesting finding, however, is that private placements are completed at a considerable discount to the firm's fair market value, as a compensation to investors. Company value also appears to be related to the type of investor, with passive investors being negatively correlated to the firm's worth.

Also, Davydov, Nikkinen, and Vähämaa (2014) examine the relationship between debt financing and the stock performance of a firm, in an emerging market setting, which is different to that in developed markets, in the sense that the corporate governance practices are different. Specifically, the authors examine whether resorting to financing through public debt, or issuing corporate bonds have the most impact on a company's stock returns. They use a comprehensive sample of companies, within the Russian markets and cross-sectional panel regressions, and find that public debt results in negative valuation on the firm. Corporate

debt, on the other hand, is associated with better monitoring, and thus lower adverse selection and moral hazard problems. In that sense, they conclude that, all other forms of debt financing are more efficient than public debt in terms of positive valuations.

Valuations also seem to be affected by a great number of other factors that can be seen in the relative literature. Firstly, Chauvin and Hirschey (1994) suggest that intangible assets, with goodwill holding a prestigious position among them, had drawn much attention by both academics and practitioners. Originally, it was highly important to understand and explain how intangible assets should be valued, and how their worth could be estimated under the relevant accounting standards. The consensus was, that at the time, there was a tendency to underestimate the economic importance of the intangible assets. Later (as shown in the study above) intangible assets were restricted to a small group of specific characteristics, such as trademarks and copyrights, while other important ones such as reputation, human capital's worth and others, were viewed as obsolete, and therefore ignored during valuations.

The phenomenon described above, can explain the disparities between book and market values, in a great number of firms. It has been made clear, that investors estimate the economic worth of an intangible asset, which is more than it shows in the accounting records. Specifically, goodwill, accounted for a large part of the firms' value, and therefore, within the boundaries of this study, goodwill is used as a proxy for all other intangible assets' value to the firm. There also seems to be a connection between goodwill and other company-specific traits. The empirical findings indicate that R&D and advertising are major determinants in the creation of value, through goodwill. More importantly, goodwill is associated with positive market-value and is considered by investors as an indicator of longevity and prosperity of the firm. Moreover, Beatty and Weber (2006) identify a relationship between goodwill and the fair value of a firm, as they explore how SFAS 142, on Goodwill and other Intangible Assets, is being implemented by managers and how it affects their accounting choices. The results suggest that, firms with a high-risk profile are prone to taking on the Standard's suggested write off, and the size of it is equivalent to the market's response to future damages realized.

R&D disclosure is also the centerpiece of the study by Branch and Chichirau (2010) who explore the signaling that substantial R&D expenditures emit to potential investors, and how those signals affect the risk premium required by them. The authors suggest that R&D events are not deemed as significant by investors, mainly due to the unavailability of further information on the research itself and what results it may bear for the company but also due to the misleading effect that the expenditures, for R&D, have on the company's balance sheet. Therefore, stock prices of R&D firms bear a significantly higher risk premium. Moreover, the authors imply that it falls under the management's discretion to release information regarding pending research results, in an attempt to increase their returns. To determine the effect of the R&D of a company on its stock returns and premia, the authors use the patents and patent citations from the NBER's patent database and find that correctly evaluating on-going research can result in possible exploitation of risk premia, and thus positive returns on research intensive companies.

When looking at operational diversification, Lin and Dongwei (2008) argue that there are conflicting opinions on diversification in the literature. One side supporting that it results in a series of benefits for both the company and the investors, while the other side supports the notion that it harms the company's value, mainly due to a number of factors related to CEOs and their motives. To provide some insight on this topic, the authors examine a sample of Chinese privatized state-owned firms, which were previously publicly listed. China provides us with a unique field to examine the effect of diversification on company value and performance, since its market is developing and thus separated from other countries' markets, which results in a higher level of information asymmetry. Moreover, it is suggested that creating an internal capital market can lead to an increase in the company's value, while there are no spillover effects from other countries discounts. Additionally, Chinese firms are characterized by a complex ownership structure, which might influence diversification strategies negatively. The study findings point to a U-shaped linkage between ownership concentration and company value, with the presence of the government owner to further deteriorate diversification strategies, and finally the decision to diversify based on specific company characteristics, is associated with a higher valuation.

A series of research papers has also explored the link between dividend ratios and cash flows, and the potential for growth, and how those are related to company value. Cochrane (2011) surveys and compares prevailing theories in modern asset-pricing. He notes that variation in price-dividend ratios is linked to that of future cash flows. Modern theories, however, suggest that price-dividend variation is associated with discount rates and their variation. He concludes that research has been inconclusive as to what causes the variation in discount rates and predicts that discount rate analysis might lead us to revisiting pricing theories as well as the way that cash flows are being generated. In a previous study, Cochrane (2008) also argues that, for the dividend-price ratios, to have an observed variation, dividend yield should be measurable, contrary to stock returns. He theorizes that, dividend growth should be able to explain the variation in dividend yields, however his findings suggest that there is lack of forecast ability when it comes to dividends. He also finds that the variation in price-dividend ratios can be explained by that of expected returns, and that excess returns vary as much as their average levels.

Furthermore, in a study by Garrett and Priestley (2012) an alternative model is created, in an attempt to show that dividend yield growth can be accurately predicted. Their aim is to explain the significance of cash flow news on the aforementioned predictability. They find that dividend growth can be explained if management choices, regarding dividends, are taken into consideration. They also discover that dividend payments are completed in conjunction with earnings and stock prices. The high correlation between those three elements allows them to accurately predict dividend growth.

Finally, the composition of the board as well as management's investment stance can be attributed to a more positive or negative valuation. Specifically, Bauguess, Moeller, Schlingemann, and Zutter (2009) examine the relationship between the number of internal and outside managing directors in a firm and company performance in the form of returns. They use data from the SEC on a pre-acquisition announcement period, for the target companies, and they categorize the ownership into active and passive, with regards to whether the people involved with the management of the company, have board representation or not. Then they proceed to determine the explanatory power of ownership on the

company's returns and premiums, prior to the acquisition. Their findings indicate that there is a strong positive relationship between inside ownership and abnormal returns, while outside ownership is related negatively to abnormal returns.

2.5.2.1 Earnings Management

Valuation analysis carried out by analysts (and investors) is also based on another very important metric, that of earnings quality. Good quality earnings as Dechow, Ge, and Schrand (2010) explain, the quality of earnings is determined by their ability to convey information about a company's financial performance in regards to a specific decision making process by a person interested in that firm, can be an indication of a healthy business and as such can gain a higher value for that company. That is the reason, managers resort to earnings manipulation, and therefore some part of the research has shifted towards determining the link between earnings quality and value.

An early study of Chaney and Lewis (1995) explains that directors tend to manipulate accounting earnings, in an effort to signal a healthy company with great prospects to potential investors. However, the authors are concerned with the question of whether the manipulation is the result of an artificial value increase attempt on behalf of the companies. For that reason, they create an asymmetric information theoretical model, in which they include high and low value firms, and the status of the firm is determined by the ability of the firm to generate positive earnings, with high value firms being trusted by investors to keep up with those levels of earnings in the future. To control for the credibility of the signaling of the firm's value they also include the assumption that over-reporting the earnings will result in a higher cost for the firm, in the form of higher corporate taxes. Their results indicate that there is a high correlation between the earnings reported and the tax that is imposed on those earnings, with high value firms being keener on paying the additional taxes, while low value firms were weighing the taxes as more important, than the valuation itself. In the study by Eng, Sun, and Vichitsarawong (2013), earnings is put against book value, as a measure of

a more accurate valuation of a company under different accounting frameworks. They use a comprehensive sample, with companies from five Asian countries, which are also listed in the US. The results indicate that book value is the prevalent measure for the domestic markets, for the period under examination, as it is more informative and less susceptible to manipulation. The results for the overseas companies are similar, up to 2007, when earnings become more reliant as a valuation method. The authors also note that companies that comply with the IFRS are also more related to earnings, while book value is more associated with the US GAAP.

Moreover, Gao and Zhang (2015) explore the connection between earnings smoothing and the valuation of a company. They state that earnings management is a common practice in firms, although they suggest that the empirical evidence on the matter is at best scarce, and it revolves around the fact that smoothing may misrepresent the actual profitability of the company, however it is also used by managers as a means to emit more value-related information to the public. This is even more evident for companies that engage in corporate social responsibility (CSR) practices, since those practices signal a higher moral standard for the company that uses them, and thus a lower chance of manipulating its earnings. To that end the authors use smoothing with the use of total and discretionary accruals, as well as operating cash flow smoothing. Their results confirm their initial notion that firms which engaged in CSR are involved in less earnings manipulation practices, while the CSR itself seems to improve the company's performance, and simultaneously makes the earnings reports more responsive to the company's actual worth.

2.5.2.2 Risk Management as a Value Factor

Proper risk management practices can be a good indication of how the firm is prepared to face any potential downsides regarding its own operations. For that reason, it can be viewed as a factor that increases the value of the firm. Guay (1999) examines how the financial reporting rules highlight a company's need to incorporate derivatives in their portfolios, and how the reporting of them affects the firm's risk exposures,

and subsequently its value. He finds that derivatives when used, for hedging purposes and not speculation, reduced the overall risk of the company.

Following that idea, Allayannis and Weston (2001) suggest that the Modigliani – Miller theorem deems risk management as of low importance to the firm, since shareholders are well diversified on their own. On the other hand, modern theories (Jorion, 2000), indicate that hedging might increase the company's value. For that reason, the authors, examine whether the implementation of a derivatives strategy results in increased company market value. They argue that companies that are neither exposed to foreign exchange risk or hedge, should not trade at a discount. Delving further into this idea, a possible relationship between various firm characteristics and the premium discussed above is explored, with the results pointing to a strong positive relation between increased firm value and the use of currency hedging.

Furthermore, Roll, Schwartz, and Subrahmanyam (2009) remark on the importance of options and the growth of their markets, over recent years, and suggest that there is an unseen linkage between these derivatives and their underlying assets' markets. Prior research suggested that the adoption of such instruments, might provide for a higher efficiency within the other markets, both in terms of prices and information communication between firms and potential investors, while simultaneously allowing traders to assume more leveraged positions without taking up higher risk. Moreover, it is pointed out that options decrease information asymmetry in stock markets, as this type of trading seems to contain more information regarding the expected movement of the stocks' price. This idea suggests that options may act as a value-increasing component, as lower information asymmetry implies lower risk associated with an investment. It is found that a higher level of options trading is related to an augmented valuation of the firm, better future financial performance, as well as, a link is discovered between higher valuation and firms with low analyst coverage, that are involved in increased options trading.

A concluding testament to the usefulness of risk management practices (in the form of the usage of the various hedging instruments), is the study of Chaplinsky and Haushalter (2010), who argue that the risk associated with a private investment in equity, as perceived by the investor, can be shown in the financial

contracts. Clauses that involve contingent claims are used, instead of a higher discount rate, when private investors are buying blocks of stock, as well as terms of control transfer to the investors. The idea of contingent claims, that can have the form of an option, is that the investors will be able to retrieve a part of the value invested, regardless of the outcome of the investment. Moreover, the authors document that the bargaining power of the stock's issuer, becomes more limited as the financing options become scarcer, and other problems arise such as, but not limited to, poor performance history for the stock, unclear investment intentions from the managers, as well as, negative cash flow creations. They also suggest that such actions are common, since in this way asymmetric information, or moral hazard problems are mitigated.

2.5.3 Value factors in Private markets

As discussed above private companies are plagued by the limited amount of available information, which in turn affects the way that valuations may be done, since basic elements like the Beta and standard deviations cannot be calculated, due to the lack of price and other historical data. A study by Chen, Dyl, Jiang, and Juneja (2015) examines the key elements for the discount rate in private equity transactions. They classify these into three basic categories, namely risk, illiquidity and marketability. Their findings suggest that risk and marketability are key components of the private placements throughout their sample period. However, liquidity (how easily an asset can be traded and is measured by the transaction costs for the trading), as explained by Amihud, Amihud, Mendelson, and Mendelson (1988), seems to be a major element of the discount rate prior to 2003, while marketability (how quickly the asset can change hands, without accounting for its selling price and whether there is a market for it) is after 2003. The authors attribute these phenomena to the changes in the market's microstructure, and to the relationship of an asset's liquidity to the higher volume of trading.

One of the other key points in public businesses is the separation between management and ownership of the company. However, that is not the case for the private companies, in which the line between manager

and owner is often blurred, and the owners are usually heavily invested in their businesses. This creates all sorts of implications, both in the sense of the lack of diversification for the owners due to the binding of their own wealth in one asset, but also for the companies themselves and the mixing of salaries and other personal expenses with corporate expenses (Damodaran, 2012).

Additionally, and in the terms of personal wealth investment within the private companies, Abudy et al., (2016) argue that the owners of private firms devote a large portion of their own capital in their firm, and thus are more exposed to that firm's idiosyncratic risk. Due to that reason and as already noted, investors in such firms, require a higher return, than their diversified counterparts, and subsequently this fact is reflected in the higher cost of equity that private firms face. For that reason, the authors introduce a methodology that is related to holdings in private firms, and the lack of an available market for their trade. They use what they refer to as "private state prices"[14], which are modified to account for the "non-marketability", as they characterize the effect, and derive from them, the discount factors which enables them to calculate the cost of capital of private companies, by comparing them to, levered and unlevered, firms that trade in active stock markets. Their findings show that the cost of equity for private firms, as well as the premium they are required to pay to investors, to compensate for the lack of marketability is an increasing function of the firm's asset risk and the level of non-diversification of the investors. Both levered and unlevered private firms display a higher cost of capital compared to public ones, with leveraged private firms being even more costly to their investors than public ones. Finally, the authors state that increase in tax rates affect private firm return premiums negatively, and that this effect is negatively affected by higher leverage.

The literature has also pointed to other factors as determinants for value in the private company setting. Initially, Gonenc, Hermes, and van Sinderen (2013), state that private firms are more often the target of acquisitions, than their public counterparts, with the bidder's returns on private firms, being significantly

[14] Those are essentially the prices for marketable securities adjusted, with a premium to the discount rate, for the fact that these investors have tied part of their wealth to non-marketable securities.

positive and higher than those of the equivalent public ones, with the literature focusing mainly on factors such as the size of the acquired firms, the marketability and liquidity of the shares, uncertainty concerning the accuracy of the company's valuation and the legislative framework, providing protection to investors in the country that the acquired firm operates. The authors set off to consider the ownership structure of the acquired firms, with regards to them being family owned. They initially support the notion that family controlled private firms are more expensive for buyers, since a significant premium must be paid to them, in order to convince the family to give up the ownership of the firm. This premium is reduced when the payment receives the form of shares, instead of other forms of payment. To that end, the authors find a significant reduction in the bidder's returns when the target firms are family owned.

On the family ownership but on a different setting, Astrachan and Jaskiewicz (2008) offer an expanded definition as to what constitutes "value" to a family business owner, by arguing that it includes both financial and emotional components and that like financial components, nonfinancial considerations add to and detract from the business's value from the owner's perspective. They present a new valuation formula that addresses, from an owner's perspective, financial and nonfinancial (emotional) returns and how they affect total business value, which they consider as an expression of business utility for the owner. According to them, their approach helps clarify the acceptable returns that family business owners desire from new ventures or acquisitions.

Another set of determinants is identified by Hespenheide and Koehler (2013). They report that, what ultimately gives a business value, is the level of information that the company discloses to its investors, as it reduces the risk perception and consequently increases value from the present value through the use of the reduced risk adjusted cost of equity. Specifically, they name environmental, social and governance disclosure as positive signals to investors, and a way for the firm to create value. Additionally, Pereiro (2001), develops a fundamentals-based valuation model to examine how valuations of private enterprises in Latin America are affected, by the country specific risk. His findings indicate that investors pay little attention to the small size and illiquidity factors, and the Betas that they use do not account for cross-border

synergies. Sabal (2004) supports this finding and proposes a course of action, to adjust for country risk, when performing valuations in emerging markets. Also, De Franco, Gavious, Jin, and Richardson (2011) suggest that a possible explanation for a reduced discount rate on a private firm, is whether this firm uses one of the Big4 auditing companies. The authors explain that the reduced discount rate is related to the higher information quality the buyer is facing.

2.5.4 Size as a discount counterweight

In the build-up method section of the literature review, we mentioned the size premium (or small-cap premium), that businesses have to pay due to their small size. This problem is pertinent in private businesses as the vast majority of them are small local enterprises, which are considered highly illiquid, and therefore are "taxed" during the valuation process with a higher discount rate (or risk premium). We feel that it is important to determine a framework of reference, as to how larger size helps mitigate the illiquidity and how this is tied to the focus of this thesis, namely the discount rates of private enterprises.

As shown in Paglia and Harjoto (2010), there are certain difficulties associated with private businesses' valuation, as there is a lack of data availability. The inability to easily sell or buy private company ownership rights, makes the adjustment of their value, through a discount, a necessity. While previous studies have mainly focused on public companies, this study accounts for the lack of marketability through the use of public companies' data, by matching them with their private counterparts. The results suggest a difference in magnitudes of the discounts, as well as the discrepancies between the various sectors, with the professional services sector having the highest and the healthcare the lowest discount. Size is an important factor when valuing private firms, as it negatively associated with discounts for the lack of marketability.

Another study on the private companies' marketability is that of Bajaj, Denis, Ferris, and Sarin (2001) who explore and compare the methods for estimating the lack of marketability for closely held companies. They use a dataset from the 1990s to compare the three approaches, most commonly used when deciding on a

proper valuation discount for a private firm (IPO approach, restricted stock approach and acquisition approach). They also come up with a large number of possible factors, which affect the discount rate, such as, the Altman bankruptcy measure Z-score of the issuing company, the number of shares issued, the standard deviation of the firm's returns and whether the issue is registered. Herrmann and Richter (2003) attempt to estimate the price of untraded equity investments. They use both American and European firm data, and identify a set of market multiples, based on a risk-neutral valuation model. They conclude that a selection of comparable assets bases on the aforementioned multiples is superior to that of a selection based on industry codes.

Furthermore, Barenbaum, Schubert, and Garcia (2015) attempt to determine the fair value of a closely held company, through the marketability discount associated with such an investment. They suggest that the usage of put options as proxies for the discount, is faulty in the sense that the upside potential of the said option, is inherent to its price. Therefore, the authors, suggest that the safest method to ensure a fair marketability discount on a private company's transaction is performed by a loan and an equity collar. This approach, as they say, reduces the discount for a potential lack of marketability.

The idea about size being of high importance to the private firm valuation is also supported by the study of Comment (2010) who provides evidence on the strong empirical relationship between size and liquidity, as he uses DCF analysis to various subsamples of firms, based on company size, to examine the effect of applying extra illiquidity constraints on his sample firms. A key point of this study is the use of fairness-opinion valuations, which are in practice valuations that instead of an absolute final value for the firm provide investors with a range of values, within which the actual price can be found depending on whether the process is performed for an M&A or an IPO. It is reported that the smaller a company is, the greater the effective size premium that is applied on the discount rate. These findings are consistent with Damodaran (2012), who further attests to this notion.

To expand further on this discussion, private firms are valued similarly to public ones, with a series of constraints that are linked to illiquidity control and non-diversification options, an investor expects to

extract value from them through the cash flow they will generate in the future and potentially through the terminal value the vested interests can be sold for. Interests in larger companies (with a plethora of assets), are much easier to be sold, as more assets suggest a healthier company, as well as a higher probability of reclaiming the original investment in the case of liquidation (Damodaran, 2012).

The idea of size being a counterweight to illiquidity can also be found in other topics related to private enterprises. Buchner (2016) explains that since private equity is characterized by high illiquidity, accurately measuring its value is very challenging and is usually done through its observable cash flows. The methods used most commonly are the internal rate of return, total value to paid-in capital and the public market equivalent. However, from those three only the last one can be used to most accurately predict the performance of the equity in question, with it being flawed as well, in the sense that it cannot take more risk factors into consideration and due to the log-utility assumption. Therefore, the author of this paper, is merging public market equivalent methods with that of the CAPM and starts by estimating the value of venture capital using the CAPM, which he further enhances by incorporating the Fama-French three factor model, and finally by adding a traded liquidity factor. The results indicate that there is a striking resemblance of the venture capital returns to those of small growth stocks, and that the traded liquidity factor's effect on those returns is barely existent.

2.6 Private Companies

The topics we have covered so far apply to all companies, but research so far has been primarily focused on public enterprises, although private companies are the prevalent enterprise type around the globe. For instance, in USA private firms employed more than 84% of the total workers[15] in 2013. They have, however, some distinct characteristics that separate them from their public counterparts, which explain up to a point why this area has been so heavily under researched. Firstly, a private company is unable to publicly

[15] Source: US Congress Research Service (https://fas.org/sgp/crs/misc/R41897.pdf)

advertise or issue shares, which leads to a “close” ownership structure and a lack of freely trading shares. Management is primarily done by the shareholders (owners), which leads to reduced or no oversight on the management. On the other hand, the shareholders have tied all their wealth with the business and therefore they are burdened with the full risk of the company as they are unable to diversify that risk away.

Damodaran (2012), explains that private enterprises vary in size and display a great number of similarities with their public counterparts, however there are some differences that set them apart. Initially, the accounting and financial standards, that govern private businesses are far less prescriptive and more relaxed than those of publicly listed firms. The loosened regulatory framework leads to less timely and available information on private businesses, in the form of data availability both throughout, but also within accounting periods.

As has been established already, available historical data, is what drives the ability to estimate a price for a company’s equity. The lack of data, and consequently the lack of an accurate price on private equity, increases the cost of equity for private businesses, and thus the cost of equity of private firms, as they create a gap of knowledge between the firms and potential investors (Damodaran, 2005). This reduced disclosure framework, under which private companies operate, and subsequently all the transactions related to them, are characterized by information asymmetry, with manager-owners of the company having a better knowledge on the financial status of their companies than any other investor. Although this problem is pertinent to public companies, it is amplified in private enterprises, as there is no readily available price to reflect the information concerning them. Additionally, as the owners act as executives in most cases in private firms, they have the ability to “steer” the company to whatever direction they deem correct, without any outside control.

Adverse selection is the end-product of information asymmetry. It occurs when one or more counterparties in a potential transaction, have an information advantage over the other participants in that transaction. In the lack of available accurate information, management, for example, may be tempted to manipulate the various financial figures of the company, and thus inflate the value of the firm. In these situations, markets

operate in a less efficient manner, since proper reporting is that process that ensures a fair price for any asset, through a reliable transformation of inside to outside information. These results are also strengthened by the research of Damodaran (2005), who explains that, adverse selection problems come into the forefront, by the different motives investors have when trading an asset. Information asymmetry problems are responsible for the higher spreads that are charged by market makers, as they assume that they bear the additional risk of trading against better informed investors. The study of Bade and Hirth, (2016), further attests to this idea, as they point towards the connection between the liquidity in an asset and the information that is available around it (reduced liquidity is connected to reduced information availability).

To illustrate the importance of information and its dissemination, throughout the markets, and how it seems to affect the decision-making process of managers, a series of studies, starting with Chen and Zhao (2009) is explored, who suggest that the variance of asset returns, can be explained by cash flow and discount rate news, with the latter representing investor's risk aversion and overall sentiment. Their results are similar to those of Cohen and Lou (2012) who explore the effect of the dissemination of the same piece of new information has on the updating of stock prices. They simultaneously examine what parts of the information integration process are those responsible for the fast and full incorporation of said information to be fully evaluated by market participants. They find that due to the limitations that investors face, both in capital and information processing, elaborate information filtering procedures (or less complicated processing structures) can result in a delay into incorporating company-related news in their relative stock prices.

It is evident that the reduced information dissemination, is a major characteristic of private companies, and is one of the reasons as to why they are faced with reduced liquidity. This problem has been highlighted throughout the literature in studies such as that of Rijken, Booij, and Buckley (1999), who examine the differences in the private and public enterprises in the UK. Their results are indicative of the differences between the two, as they report a discount of about 40% more in private companies, which is however significantly reduced, when correcting for size, which is expected as listed companies have higher

valuations than private ones. It is evident that the higher discount is a form of compensation for the uncertainty associated with private firms, due to the lack of relative information (Chung and Hwang, 2010).

So far, the focus of this section has been to give prominence to those attributes of private businesses, which separate them from the public ones. The analysis, however, would be incomplete without pointing out their numerous similarities. One thing the relative literature has highlighted is the similarities between publicly listed and private enterprises. Specifically, studies such as those of Cooper and Priestley (2016), and Asker et al. (2015), while others such as the one by Elnathan, Gavious, and Hauser (2010), deviates from the rest of literature and focuses on the differences in the valuation of public and private enterprises. Their findings suggest that appraisers of private enterprises seem to rely mostly on earnings reports, while this does not hold true for public companies, as appraisers focus on non-financial statement sources, such as the accruals. Furthermore, they dismiss the usefulness of the appraisers, as they suggest that their interests and those of the investors who hire them are aligned, and thusly the valuations are biased. Regardless of the last study noted, the consensus in the literature is that, public and private companies share many similar characteristics and thus the factors that affect their valuations coincide in many cases, as will become clear in the sections to follow.

The last point we want to make regarding the differences and similarities of private and public businesses is related to the valuation of these companies and can be found in the studies such as the that by Elnathan et al. (2010), who differentiate between the different types of valuations (depending on whether they are performed on private or public businesses), under the premises of who is conducting them. One category, named as public valuation, is performed by analysts in investment banks and brokerage firms, and has been put under scrutiny multiple times, mainly due to the conflicts of interests that these analysts may face. On the other hand, there are the expert valuations, or private valuations, that are conducted by experts, as a means to determine the value of a firm under buyout or acquisition. Elnathan et al. (2010) focus on the differences in those types of valuations, using an event study methodology, with results indicating that analysts' valuations result in a net 20% increased value, with investors receiving these reports in a skeptical

manner. The investors are also cautious over private valuations, as well, since they might believe that private company transactions might be including a control premium, or that the experts conducting it are biased towards their employer. Further results do not support or disprove the theory that private valuations are more analytically detailed than their public counterparts. The same notion that any valuation attempt should be viewed through the prism of the purposes it is conducted for is also a point raised by others throughout the literature[16].

2.6.1 Other Theoretical Models on Private Companies

In the previous section we began the analysis, by explaining some problems that are created by investing in private enterprises. As we explained among others, private businesses cannot issue and advertise shares or list shares. This limitation was the driving force in the creation of the Accounting Betas. As private enterprises are not publicly traded, they have no available price history, with which investors can calculate a Beta. The original idea, and one that is still implemented by many investors, is to approximate private companies' data by using comparable public ones (Dukes, Bowlin, and Ma, 1996) or industry Betas. Other approaches, as the one we will discuss in this section, namely the accounting Betas, attempt to establish the risk associated with a private company, by using other information that is available. One must keep in mind though, that private businesses operate in a reduced disclosure environment. They produce annual financials, and subsequently effective assessment of future risk is difficult.

The second topic we will discuss in this section, but we have already mentioned in the introduction of the literature review, is the total Beta, which is a measure parallel to the CAPM's Beta, with the differentiation that it is targeted towards private enterprises, that do not have historical data to calculate their actual Betas.

[16] See for example the books of Damodaran (2002), Mercer (2008), Penman (2010).

This measure was proposed by Damodaran and has gained significant support over the past few years. It is also a measure that we propose to use in this thesis.

2.6.1.1 Accounting Betas

In order to estimate the cost of equity, the various methodologies developed above, use different metrics, with Beta being estimated in the CAPM with historical prices for both the stock and the market that the company operates in. The same input is also required by the multifactor models. However, historical price data are non-existent for private companies. In this case researchers and academics have shifted towards other measures of the Beta. One of those methods is that of the accounting Betas, which are Betas calculated through the use of variables from the companies' financial statements. As Damodaran (2002) indicates:

$$\Delta Earnings_{private\ firm} = a + b\Delta Earnings_{S\&P\ 500} \tag{2.10}$$

The earnings of the enterprise are used, as they are more often than not, available to appraisers, while the availability of other data is lacking. The accounting Beta is obtained by regressing the differences in the private companies accounting earnings, with the earnings from an equity index, as is shown in formula (2.10) above.

Some early work on the topic of accounting Betas, was carried out by Bowman (1979), who was among the first to try and relate systematic risk to the accounting Betas, in a seminal study, by using variations of a model on leverage that was developed earlier by Hamada (1972). Specifically, the author explains that in order for the relationship between leverage (expressed by the debt-to-equity ratio) and the Beta to become apparent an additional hypothesis needs to be added to the original hypotheses of the CAPM. He suggests that both firms and individual investors face the same borrowing and lending conditions and based on that he formulates the expected returns as a function of leverage to which he later incorporates the probability

of the company's default and taxes (spurred by the Modigliani – Miller Theorem). Following the previous logic (and model), he also attempts to mathematically establish a relationship between the market Beta and other variables, such as the accounting Beta (which in this case is defined as the covariance between corporate and market earnings and it will not be available for private enterprises), earnings variance, size, company growth and finally dividends.

The point we made at the end of Hamada's paper (that systematic risk is partially reflected in the company's leverage) has also been explored, to some degree, together with the importance of a specific set of Betas based on accounting determinants, in the paper of Mandelker and Rhee (1984). The authors decomposed Hamada's leverage into its operating and financial components and set to determine how each of those affect the market risk's measure, namely the Beta, by their respective risk counterparts. Furthermore, they examined, how these two types of leverage interact with each other to produce the mix that affects a quarter of the market risk (as Hamada has found). The results from the cross-sectional regression analysis on the manufacturing firms' sample, reveal the positive contribution of both operating and financial leverage on the Beta, or to express it in simpler terms, the higher the degree of leverage the higher the risk of the stock. Also, and in accordance with the negative correlation between operating and financial leverage that one would expect, the results reveal that riskier stocks are associated with firms that actively change the mix between those two types of leverage in order to reduce their overall risk levels.

The paper of Mensah (1992) also drew inspiration from the aforementioned studies of Hamada (1972) and Mandelker and Rhee (1984). The argument set forth by this paper, in favor of the accounting Betas used in the literature, was that although the market Betas may not be driven directly by accounting data, it is these kinds of data that reflect the fundamental economic attributes that affect the Betas but are not observable in a direct manner. For that reason, the accounting variables are needed to act as their proxies. In that sense, the author attempts to extend the literature, by developing a model that incorporates a strategic decision-making component to the operating and financing strategic mechanisms of a firm, as those are reflected by its systematic risk. It is also noteworthy that principal component analysis is employed in this paper, as a

method to eliminate the effects of multicollinearity, which is also the methodology used in this thesis. The results support the importance of the accounting Betas as accurate substitutes of the market Betas. Moreover, the findings suggest a stronger relationship of the cyclicality in the earnings and cash flows of a firm relative to its competitors with the market Betas, than the leverage components of the firm. As the author suggests however, the results might be affected by the assumption of linear relationships between the market Beta and the financial statements' variables.

Another study, that further attests to the results of Mensah (1992), was carried out by Almisher and Kish, (2000), who examined the relationship between market and accounting Betas. They explain the significance of the accounting Betas in the determination of the risk associated with private enterprises and how their findings support the idea that accounting Betas can act as substitutes for the market Betas in the IPO process, as they find that the proxies they used for the market Betas (that were formed through the use of accounting variables), suggest a strong relationship between market risk and risk measures drawn from accounting variables. They also argued for the accounting Beta as an appropriate risk measure, by focusing on the criticism over the alternative three-factor model, proposed by Fama and French, as the extra factors were mostly data driven rather than actual risk factors and were lacking in theoretical background. These results have been disputed however, by a later study of St.-Pierre and Bahri (2006), who focus on small and medium enterprises. They indicate that although accounting Betas might work for public companies, in the case of the private ones, they are lacking in recording all the risks associated with small private firms, mainly because they are based on ex post financial information. For this reason, they advocate towards adding nonfinancial information, such as the market structure competitors, new technologies and innovation to the risk factors.

The importance of the Accounting Betas, as a means of approximating firm value can be highlighted by the continuing interest in the topic. Penman (2010), suggests for instance, that accounting information is the basis for all statistical or mathematical modeling that is prevalent in valuation research, as all betas that come from such models have their basis, on accounting variables. This idea is pervasive in the literature,

as exemplified in studies such as those of Joos et al. (2016) and an earlier study of Thomas and Zhang (2009), that both examine the quality of the information from accounting variables and how financial risk can be assessed based on the betas derived from those variables. In most occasions, accounting betas are used as a proxy, or more accurately, as a substitute of the CAPM beta, when the latter is not available (which is the case with private companies also). This practice has seen some criticism (Sarmiento-Sabogal and Sadeghi, 2014), as the correlation between these two measures, which is used to evaluate the effectiveness of the accounting betas, tends to overestimate the market risk. Other critics followed the idea that, as CAPM's beta can be improved by the inclusion of other characteristics in the model, so could accounting betas, which led to the introduction of ideas such as the earnings – consumption beta (Bergeron et. al., 2018).

Specifically for private entities, Damodaran (1999a, 2012), referred to the accounting Betas on multiple occasions, as a means of regressing the earnings of the private firms against the benchmark market's returns. However, this method is faced with two major considerations: the first one is the limited data, as private companies are not required to report earnings on a regular semi-periodic basis, and the second and perhaps more important is the possibility that earnings might be manipulated. The next methodology is that of the fundamental Betas. This methodology resembles the multiples version of company valuation, as several specific variables are used[17] to create a framework with which a Beta of public company is estimated and then the same coefficients estimated in the regression, can be used for a similar private company. This approach has the problem of the input quality of the variables. Poorly chosen or otherwise insufficiently specified variables can lead to overall weak results. The last methodology is that of the bottom-up Beta. With this approach, we again estimate the private company's Beta through a public company approximation. Specifically, with this method, one adds up all the risks, that a company faces when conducting its business. This approach also requires the leverage adjustment proposed by Hamada, which

[17] These variables are identified as Damodaran explains, in the seminal studies of Beaver, Kettler, and Scholes (1970) and later on from Rosenberg and Guy (1976).

is somewhat difficult to find for private companies, and as such we assume that the private firm's leverage is similar to the average of the industry it operates in.

2.6.1.2 Total Beta

At this point it is critical to stress again the problem of the non-diversification of the investors in private businesses. As we have seen, the owners of private enterprises, are usually fully invested in their firms, and subsequently they are not diversified, and subsequently cannot reduce company-specific risk. This is contrary to the classical financial theory, where investors are thought to be well diversified and traditional Betas are a good measure of systematic risk, as new stock is added to portfolios. Betas, on the other hand, are a measure of the sensitivity of the addition of an asset to a portfolio, and therefore are not appropriate for the investors in private firms, since they will not properly depict the unsystematic risk those investors face. Damodaran (1999a), suggested for that reason an alternative version of the Beta, the Total Beta, defined as:

$$Total\ Beta\ = \frac{Market\ beta}{corr(C, M)} \tag{2.11}$$

where C stands for the private company and M for the market, to account for the adjustment needed. For this metric, Damodaran, indicates the correlation of the private company's stock (he calculates the market and correlation of the private markets with the help of similar comparable public companies) and the market the company operates in, as the weight that determines the size of the Total Beta. He explains that higher correlations will lead to lower Total Betas, as a lower correlation suggests lower unsystematic risk, and that this adjustment should be used in relation to the reason the valuation is being conducted for. For example, there would be no point in making this adjustment if the valuation was completed for an IPO because as a listed company the portfolio approach is appropriate and unsystematic risk can be diversified away.

Furthermore, Butler (2010) explains that estimating a relevant cost of equity for a closely-held firm is a highly difficult task, mainly because there are no specific rules as to how this type of company should be approached, and thus experts approximate them through data from the public stock market. They do this rather than employ a subjective addition for specific risk, which was the practice in the past (see James H. Schilt's table (Table 3) presented below). Total Beta was introduced to counter this problem, as it is used to estimate the total risk, and therefore should be used by practitioners, for them to estimate their discount rate more accurately.

Schilt's Risk Premium for Discounting Projected Income Streams

Category	Description	Risk Premium
1	Established businesses with a strong trade position, well financed, with depth in management, whose past earnings have been stable and whose future is highly predictable.	**6-10%**
2	Established businesses in a more competitive industry that are well financed, have depth in management, have stable past earnings and whose future is fairly predictable.	**11-15%**
3	Businesses in a highly competitive industry that require little capital to enter, no management depth, a high element of risk and whose past record may be good.	**16-20%**
4		

	Small businesses that depend upon the special skill of one or two people. Larger established businesses that are highly cyclical in nature. In both cases, future earnings may be expected to deviate widely from projections.	**21-25%**
5	Small "one person" businesses of a personal services nature, in which the transferability of the income stream is in question.	**26-30%**

Note: "The risk premium chosen is added to the risk-free rate...." The resulting figure is the risk-adjusted capitalization rate for use in discounting the projected income stream. Because of the wide variation in the effective tax rates among companies, these pre-tax figures are designed to be used with pre-tax income.

Table 3: James H. Schilt, "Selection of Capitalization Rate – Revisited" Business Valuation Review, June 1991, p. 51. This table was drawn from NACVA's ('Chapter Six Commonly Used Methods', 2012): "*Fundamentals, Techniques & Theory: Capitalization Discount Rates"*, p. 30

It is very important to stress that this measure of company specific risk, should be used for private firms only, since portfolios that include public ones allow for full diversification of this type of risk. Butler, Schurman, and Malec (2011) also point out that Total Beta has been tested in court under the Daubert principle and has its roots in the USA in modern portfolio theory. They also note that, Total Beta produces more efficient results than CAPM Beta, when appraising private firms. More importantly, since the number of buyers matters, and buyers also try to price for all their risk, it follows that undiversified buyers do not have the power to price for all their risk, and therefore they should not be using a measure that does just that, namely the CAPM Beta.

Another, theoretical model, which was originally developed by Dohmeyer and Butler (2012), the Implied Private Company Pricing Line (IPCPL) model, is used to estimate a fair price or cost of equity for private

enterprises. The theory behind this model suggests that there is a systematic relationship in the returns of the public and private markets, given the no arbitrage opportunities between them. This relation can be exploited, according to the authors, by applying the regulatory compliance burdens, as well as the transaction costs, on the price of public companies, to determine the fair value of the equivalent private ones. Another implication of this model, as the authors suggest in a later study, is that it enables the incorporation of any illiquidity differences between the public and private company prices, within the value provided by the model. Specifically, Dohmeyer et al. (2014), based on their previous work, developed a model that allows for the estimation of the cost of capital for small capitalization private firms, and permits adjusting for differences in systematic, total and diversifiable risk, as well as liquidity and debt. They then modified this model to account for any "peculiarities" some firms may have, in terms of their fundamentals. They suggest that, the build-up method used by appraisers is lacking, mainly because the company specific risk premium is based upon rates of return that are practically non-existent in private companies. They also claim that the Implied Private Company Price Line (IPCPL) model they developed earlier, overcomes this problem, by using fair market value prices used in private company transactions.

Goodman and McLelland (2015) further develop this model. Firstly, the authors explain that most of the criticism on the model derives from the idea that since private and public companies are essentially different, they must also have different risk profiles, and subsequently they must be burdened by dissimilar prices. The authors also explain that this idea is further spurred by the fact that IPCPL establishes this relation between the private and public companies using public company data. However, the authors suggest that the IPCPL, holds under the no arbitrage theory. Then they propose an adjustment in the model, with first and second order derivatives, to correct for the risk sensitivity of the private equity's returns. They conclude, by claiming that after this adjustment the IPCPL theory holds, and thus the model can be used in private company valuation practices.

Some studies attempted to test the theories developed before. Specifically, Kasper (2010) challenges the results produced by the Total Beta concept and its application through the Butler-Pinkerton Calculator, as

he argues there are several inconsistencies, with the logic underlying them. Specifically, he mentions that although Total Beta approximates private company valuations, using public companies' data, public stock returns cannot be estimated through it. Moreover, he argues that the Capital Market Theory indicates that stocks, not held in a portfolio, exhibit no explicit relationship between their total variation and total returns, and for those within a portfolio, their returns can be estimated through the CAPM, something that is not fully reflected by the Total Beta.

Conn (2011) examines the Total Beta proposed by Damodaran, and the Total Cost of Equity (TCOE), backed up by Butler and Pinkerton, and attempts to put those methodologies to the test both in terms of efficiency and applicability to real-life valuations. His findings suggest that although Total Beta can be a valuable tool in the quest for estimating the risk profile of the firm under examination, the equity risk premium provided by this methodology, provides analysts with questionable results. However, it is pointed out that both Market Beta and Total Beta seem to eventually produce the same results, with the Total Beta exhibiting lower ex post volatility for small cap firms. On the other hand, the author claims that there is little evidence to support the most basic idea of TCOE, namely that investors require the same return as compensation, for undiversifiable risk, as they do for the systematic one.

2.7 Conclusions

In summary we can say that the first impression one gets by systematically appraising the literature is that it is a labyrinth of different ideas and different methodologies, which seems at a first glance very hard to navigate through. It becomes however much easier to follow if we look at the big picture, namely focus on the two primary literature streams that define it. The first revolves around the cross-section of the expected returns, and how those are calculated using the CAPM. This theoretical strand has devout followers even today (for example see the paper of Cosemans et al., 2016), however its original form has been amended several times by the addition of more premiums, as researchers realized that maybe there are more issues

than the CAPM takes into consideration. And this, along with other studies, with most notable those of Ross (1976) and later Fama and French (1993), spurred the development of the second strand in the literature, that of multifactor models.

Regardless of the theoretical stream however, everyone realized that the questions that needed to be answered were how and why the discount rates vary over time, as discount-rate variation became the center piece of modern valuation literature. The idea of generated cash flows in the future as the major characteristic in the estimation of asset prices, has defined the relevant literature for many decades. However, as Cochrane (2011) explains, the discount rate constituent part is far more promising in explaining asset price variation. Arguably, discount rates are a more suitable candidate in the study of asset pricing. As they vary over time, they allow us to observe how the affected asset prices fluctuate at the same time. It is also far more informative as a tool in observing any economic asset over a longer horizon, as it provides researchers with a much greater insight, as the discount rate reflects the risk associated with the asset at any point in time. This allows them to better predict how the asset's price will fluctuate during different stages of the economic cycle, as other studies have noted for example the study of Lamont, 2000, (mentioned in the EMRP section of the literature).

Moreover, discount rates can act as the link between different asset classes. This point is a recurring one throughout the literature review (and will also be brought up again in the Methodology Section). To better visualize the meaning of this, we can start by thinking of the various assets that are being valued. Sovereign debt and bond valuation is based on spreads (Amira, 2004; Güntay and Hackbarth, 2010). Stocks value is tied to dividend yields (Chan et al., 1992). Foreign exchange is defined by interest rate spreads (Booth, 1999). All these varying financial instruments reveal a pattern. Regardless of the asset class, all the risk associated with it can be reflected in the discount rates or expected returns or premiums, which describe essentially the same thing. (Cochrane, 2011) describes this as an omnipresent phenomenon and specifically, he exemplifies this by explaining how high valuations are tied to low returns regardless of the asset in question. Therefore, examining how discount rates are tied to risk and how they evolve over time, might

allow researchers to develop a unified framework on asset pricing behavior, given a set number of common characteristics (with some minor individual traits). That last remark is what gave birth to the series of questions, this thesis sets out to answer.

3. Methodology and Data

This study has multiple aims. Primarily we want to, distil, catalog and sort the risk factors incorporated within the discount rate (as this is exemplified through the lens of the P/E ratio), as they are highlighted in the related literature. This will then be used to incorporate a full and systematic evaluation and analysis of unsystematic risk and its constituent elements, which will in turn help us determine, in this contextual framework, the relationship between risk and return, and how it is established in the valuation process through the discount rates. The objective is to use quantitative testing techniques to construct a framework under which business appraisers will be able to assign an appropriately informed and scientifically defensible discount rate for private companies. Finally, we aim to highlight any differences in the way investors in the UK and US perceive the risks associated with investing in private enterprises.

The relative literature thrives with a plethora of determinants that might affect the P/E ratio, which serves as a proxy for a closely held company's valuation. Those elements may range from firm specific characteristics (Abudy et al., 2016), to other characteristics that include macroeconomic factors, for example country specific risk (Sabal, 2004), or industry related ones (Hertzel, Li, Officer, and Rodgers, 2008). Due to the number of factors though, analysts will usually have to rely on their instincts, more, than any actual established guideline. The personal beliefs and ideas of both academics and professionals, has led to great discrepancies in the ways that discount rates are calculated. This thesis aims to derive and consolidate the factors that drive the discount rates for private companies' valuations and highlight the most critically prominent among them. The main idea is to create a reference framework, under which a fair price can be effectively and systematically estimated.

Moreover, this cross-country comparative study, will allow for any potential differences between the US and UK valuation methods to be indicated. This comprehensive evaluation and critical study to produce a valid and effective framework incorporating cross country differentiation is something that has not yet been comprehensively carried out in this field, and thus constitutes the contribution to business valuation knowledge. To properly begin addressing the issue at hand however, we must first define what the key

research questions, which derive from the literature, are. Then we will link these questions with the hypotheses and explain how those will be answered through a series of tests. In order for us to do that we will need to explain what kind of data we use and how we concluded that those are the most pertinent ones to address the problems raised. We will conclude this section with the proposed methodology and the models that will be used in the thesis.

3.1 Research Questions

The Discounted Cash Flow methodology is the most commonly applied method of valuation, as Brotherson et al. (2013) among others, state. It is after all the method that all other methods have their roots in, as Damodaran (2012) argued. It requires, however, the input of various elements, namely the future cash flows a project (or investing in a firm) will create as well as the discount rate, in order to calculate the Net Present Value of said project. It is imperative, for the results to have validity and be accurate, for the input parameters to be as realistic as possible. This has initially led the theory of asset pricing to evolve around the idea of the expected cash flows of the project. However, it soon became apparent that the need to find the correct input for the discounted cash flow models could not and should not be covered by the expected cash flows but rather by how much compensation the project should provide to the investor for the risk they take when binding their wealth with it.

As is evident from the literature, many have set forth to discover what really drives the discount rates. In fact, and based on this review one would say that the potential factors that might affect it are extremely varied, as they draw elements from all the economic, financial and even the psychological spectrum. Cochrane (2011) in his survey paper refers to this phenomenon with the acute description of a “zoo of factors”. And it is a zoo indeed, as these factors can be classified in several categories, that are macroeconomic (Cooper and Priestley, 2009; Hackbarth, Miao, and Morellec, 2006; Strong, 2003), behavioral (Hirshleifer, Subrahmanyam, and Titman, 2006), financial (Bali, 2008; Fama and French, 2006; Hanna and Ready, 2005), liquidity-based (Ang and Bollen, 2010; Franzoni, Nowak, and Phalippou, 2012;

Robinson and Sensoy, 2016) and market structure-focused (Bakshi, Carr, and Wu, 2008) ones. Although Fama and French (1993), suggested that their original three-factor model was sufficient to explain the expected return variation, research in this area has not stopped evolving. Eventually, even Fama and French were convinced that there might be more factors that affected the discount rates and set forth to examine additional options (Fama and French, 2006, 2012). As the field of P/E ratio variation thrives with determinants of all forms and shapes, it led to the formulation of the first research question:

$\boldsymbol{R.Q_{.1}}$: *Which specific macroeconomic, industry and company-related factors, affect the P/E ratio, and subsequently the valuation process?*

Macroeconomic variables hold a primary place in the expected return literature, and a particular area of interest is how shocks affect the markets and how investors perceive the increased volatility signified by those. The basic idea of the CAPM model has developed, such that in recent years more complex ancillary areas have been considered for example, Kurach (2011) reports an increase in the risk premia attributed to country risk throughout the eurozone since the beginning of the financial crisis and in a later study (Kurach, 2013) finds that global factors, such as shocks, are the most important contributors to country Betas' volatility. Other studies, such as Chen et al. (2015), illustrate how private equity placements shift through various asset classes in times of distress, in order to increase their returns and how this affects the overall market liquidity. Similarly, but perhaps in a less direct way, Robinson and Sensoy (2016) examined how increased liquidity premia attract private equity, who are essentially vehicles of liquidity allocation throughout the financial sector, and how these premia have made private equity funds outperform the public markets in terms of returns. These premia reflect the increased cash-flow risk, which is expressed as the variability in the covariance between market and security returns, which rises significantly in times of uncertainty (e.g. the financial crisis of 2008). Others like Adrian et al. (2015), take a more direct approach and examine how the dot com bubble and the recent financial crisis affected the time variation of the excess returns and how investors react in anticipation of troubled periods. One can easily deduce from the above

that the study of the discount rates in times of economic distress might provide us with further insight on the behavior of investors and how their expectations on the increased volatility are reflected to the risk premia. This idea is addressed in the second research question:

$\boldsymbol{R.Q._2}$: *How do external shocks affect the variation of the P/E ratios?*

In order to answer these questions, one has to review the methodological tools used thus far in the literature, and, as Kaserer and Kraft (2003) eloquently explain, determine which statistical technique has the most powerful estimation capabilities. The literature so far, on the methodological element, has focused in its earliest iterations on identifying the relationship between the risk of investing in a firm or an asset and the returns this investment will yield through linear models, with the CAPM being the best example of that. Many objected to that idea. For example Ross (1976), with his arbitrage pricing theory, Fama and MacBeth (1973), who examined returns through the prism of the "efficient market hypothesis" and later on Fama and French (1992), with the introduction of factor analysis and the subsequent factor models they developed, and much of the academic and professional community embraced (Cochrane, 2011). There were even attempts to unify those two ideas in a single framework (Wei, 1988). Even the staunchest of the CAPM defenders realized that defending a strictly linear relationship between risk and returns leaves a lot of questions to be answered, especially since several studies (e.g. Da et al., 2012; Liu, 2006; Livingston, 2014) reported significantly increased error terms (or to put it another way started focusing on the high error terms) that were a byproduct of the CAPM regressions (Cochrane, 2011) that could not be explained by traditional linear models. That realization spurred the adoption of other methodologies to be implemented in conjunction with the CAPM (Cosemans et al., 2016), that decrease errors and thereby improve the quality of the Betas. Finally, researchers' perception also changed, in terms of the horizon that expected returns should be examined over (Cochrane, 2008), and with it also the methodologies that should be used, as well as the data, changed. As we take all of the above into consideration, we will take a different route as we

want to determine what really affects the discount rate (as this can be seen through the prism of the P/E ratio). Therefore, we pose the following question:

$\boldsymbol{R.Q_{.3}}$: *Can the P/E ratio variation over time be explained by factors that are predominantly considered to be associated with growth or high liquidity-seeking firms, or are other factors, more closely related to mature firms, more relevant to the valuation process?*

The focus of this thesis is the valuation of private firms and the next research questions are focused primarily on those and to facilitate this research, data from publicly listed company comparables will be utilized. The link between private and public enterprises is widely accepted to exist (e.g. Cooper and Priestley, 2015; Gilje and Taillard, 2016). We are going to put that link to the test to examine the relationship between private and public companies. As with public firms, the value of a private firm is determined by discounting the cash flows it is expected to generate. These cash flows need to be discounted at an appropriate rate, which includes all the non-diversifiable risks that are linked to investing in this firm. At this point, though, it is crucial to remember that although public and private firms share a great deal of similarities, the latter operate under much less transparency than the former, and are subsequently faced with a higher discount rate (Rijken et al., 1999). Despite this problem, one must also account for other things to determine the level of risk incorporated to the investment in a private firm, namely the inability of its investors to properly diversify, the illiquidity of the company's stock, the controlling interests and of course the reason for which the valuation is performed (Damodaran, 2012). Accordingly, we inquire:

$\boldsymbol{R.Q_{.4}}$: *How is the Discount Rate Function's (as this can be seen through the lens of the P/E ratio) synthesis and variation altered, when private companies are the investment targets, as the constraints imposed on these firms due to the lack of information availability and control burden them with higher premia? Also, are the characteristics that define the Discount Rate Function in public growth firms, transferrable to private enterprises?*

For the final research question, we are not so heavily motivated by the literature per se, as much as we are driven by curiosity. There has been considerable research done on investors' behavior, in regards to their expectations, (for example see the papers of Peng and Xiong (2006) or Mele (2007)) and how these rational or irrational expectations affect their decisions, based on their assessment of risk. We should not forget what we discussed in the beginning, regarding Mercer's G.R.A.P.E.S. principles, which indicate that an investor has sanity, rationality and consistency as their basic characteristics. This is the centerpiece of all the basic economic theories. The paradox however, in this strand of the literature, is that the primary notion behind it is that investors' expectations are wrong. For that reason, the focus is on the under or over-reaction of the investors to new information and how these reactions affect asset prices. Cochrane (2011), explains, however, that in reality the line between what constitutes rational and what is not is somewhat blurred. For example, in an upward economy, investors are overly optimistic, something that is reflected in asset prices, while the same assessment in a downward economy will be viewed as irrational. Behavioral finance theories on the discount rate topic are somewhat tangled, however this debate makes one wonder what constitutes a rational behavior for investors of different countries, as those are most probably going to be affected by socio-economic factors that will be different in each case. For that reason, we ask the question:

$\boldsymbol{R.Q_{.5}}$: *Should the variation in the discount rates of different markets be attributed to the different risk profiles of the various investors around the world?*

The aspiration of this thesis is to use a novel methodology (developed in the following sections), to consolidate all the factors identified in the literature and determine which are the ones that account for the greatest part of the variability in the discount rates. This will improve the understanding of how the real value of private companies should be estimated. This discussion and the research questions that were formulated, also lead to the creation of the hypotheses, which this thesis will shed light on, as those will be developed in the section that follows.

3.2 Hypotheses Development

Defining a series of research questions is, only the first step, towards examining the framework, under which discount rates for investing in private firms, can accurately be defined. To that end, we will be using a methodology, which will allow us to incorporate as many factors (the "zoo" of factors that Cochrane, 2011, referred to) identified by the related literature as possible. As our main objective is to distill these factors, however, what is of particular interest to us, is which of these variables will be more impactful to the valuation process (as this is reflected through the P/E ratio). The literature, thus far leans more favorably towards financial factors, such as the assets of a firm (Fama and French, 1997, 2007) or its debt level (Korteweg, 2010), more than any other type (for example management characteristics). These ideas lead us to our first hypothesis:

Hypothesis 1: *Variables which are derived from company fundamentals, such as Assets, WACC, EBITDA, Debt-to-Equity and Tobin's Q, will dominate the components as opposed to more qualitative variables.*

The next research question raised through the literature, was on the impact external shocks have on the valuation of private and public firms. The financial crisis of 2008, has seen considerable coverage throughout the literature (see for example the studies of Lins et.al 2013; Bertsatos et.al, 2017). Although previous shocks, such as the dot com bubble for example, spurred a series of legislative acts and perhaps shifted investors away from specific types of companies, they did not have such a great effect on the fundamentals of the firms (Kurach, 2013). As such, it is one of those shocks that appeared to have changed the way investing in growth and mature firms is done. As this might be the case, we formulate our next hypothesis:

Hypothesis 2: *The financial crisis had a severe impact on the P/E ratios and subsequently to the valuation process itself.*

The question that comes naturally for a comparative study, such as this thesis, is related to the differences between the markets, which the focus companies operate in. Intuitively, one would expect, for growth firms

to be associated with higher returns, and thusly more risk, than their mature counterparts. This idea, however is not wholeheartedly supported by relevant studies. For example Fama and French (2006), suggest that small growth firms, are not associated with higher returns than similar firms in more mature markets. Hughes et.al (2009), on the other hand support the notion that higher growth in firms is associated to higher volatility (risk). These discrepancies in the theoretical framework, as to whether or not higher growth firms should be associated with higher risk, and subsequently investors in said firms should be viewed as risk-seeking, spur us to formulate the following hypothesis:

Hypothesis 3: *Companies in growth markets, such as the AIM, are associated to higher risk than those that operate in mature markets.*

The link between public and private firms seems to be a well-established one. Several studies attest to the fact public and private firms share some common characteristics (Brav, 2009; De Franco, 2011). While our research is focused on how the valuation of private firms is affected by a series of different factors, we considered that it would be beneficial, to have a complete overview on how different types of incorporation interact with financial, accounting and managerial characteristics. We use the AIM and parts of the NASDAQ to find companies of similar characteristics (in terms of size and illiquidity, among other characteristics) with private firms as we want to add our own perspective to the relevant literature. In this way, we enable the use of specific public-companies' characteristics, for future research towards private firms. Subsequently we hypothesize the following:

Hypothesis 4: *The factors that affect the valuation of public companies, are similar to those which affect the valuation of private ones.*

To test these hypotheses, as we have previously discussed, we will use two different samples. The first will be from the AIM market in the UK, which represents the public equivalent of private firms, as the constraints faced by companies incorporated in it are similar to private ones. The second will be from the

NASDAQ in the USA, or to be more accurate from those parts of NASDAQ that match the AIM companies and provide us with a point of reference to compare the two countries.

We will, then, use these samples in a PCA process to create a new set of variables. These factors will be used in a fixed-effect regression framework, to examine how they interact with our proxy for valuation, the P/E ratio. The results from the public companies (AIM and NASDAQ), will finally be compared to those from a regression analysis between the components from the original variables and the P/E ratio, from private firms' mergers, adjusted for illiquidity. These results will also provide us with answers for the risk profile of investors in different types of companies.

3.3 Data

3.3.1 Introduction

The main focus of this thesis, as explained on several occasions throughout the thesis thus far, is the valuation of private companies. Until recently, only a handful of research has been conducted on this topic, this is something that can be mainly attributed to the difficulty of obtaining accurate data on private businesses (De Franco et al., 2011), despite the fact that private enterprises are the most common type of business entity in the world (Cooper and Priestley, 2016). Due to the lack of data, a major part of the research has used public companies as proxies for private ones. This practice is acceptable, both by academics and practitioners, not only due to the absence of a better estimate, but also due to the intuitive conclusion that private companies are exposed to the same risk factors as the public ones, with the addition of the ones previously mentioned, which result in a significant increase in their cost of capital.

The approach adopted in this thesis, will be to use public companies' data to draw conclusions on the term structure of the discount rates. We will then use data drawn from databases specialized in private companies, to examine the validity of these results. Something we need to highlight at this point is that the data for the private companies will be based upon the results of the methodology applied here. As such they will be further discussed in the results section, after the initial phase of testing has been completed. This chapter is organized as follows: We will discuss the country selection and why we chose to do a study that focuses on more than one country. We will then explain the markets selected and how the data sample was built. Finally, we will review the descriptive statistics on these original datasets.

3.3.2 Country Selection

This study aims to provide appraisers and academics with a framework that will allow them to effectively estimate the discount rates, regardless of the setting in which the valuation is conducted. In order to do that

we will need to find whether the results of this study hold in different countries, as companies that operate under a different legal and market framework might exhibit common characteristics that will serve as markers in recognizing patterns for premiums and subsequently expected returns. For example the comparative study of Aggarwal and Goodell (2011), examines the differences in cultural, legal and financial architecture in several (33 in number) developing and developed countries, to determine their impact on the ex-ante equity premia offered in these countries. Another significant example of a comparative study, which will be referred to in this part of the thesis, is the paper by Gerakos et al. (2013), that examines the effect of different market listings' regulatory frameworks on expected returns, by looking at the Alternative Investment Market (AIM) of London, and several US markets (NASDAQ, OTCBB and Pink Sheets' market).

Although we only mention these two papers as examples here, comparative studies are very common in every research field, especially when the goal of the research is to build a general theoretical framework. For that reason, we decided to focus on two distinct countries, the UK and the US.

The reasons we decided on these two specific countries are easily explained. Firstly, both the UK and the US are amongst the most developed countries in the world, evident by the fact that they are both part of the G7 and viewed as the moving force behind the global economy. A closer look at their macroeconomic elements indicates the many similarities between those countries. Starting with their GDP growth rate has been mostly in an upward trend before the financial crisis, with the US having minor corrections but in general remaining at the same levels. In 2016 the UK reported 1.6% and the US 1.8% growth rate. Another point that can be made at this point, is how fast both of those economies recovered after the financial crisis, which first hit the American market and its effects spilled over to the rest of the world with a slight, at least in the UKs case, lag[18].

[18] The link between countries and their macroeconomic variables has been explored in several studies, see for example Croux, Forni, and Reichlin (2001).

Figure 5: Comparison between the UK and US 10y Government Bond for the years 2004 – 2018 (Source: OECD[19])

Other macroeconomic indicators follow a similar pattern. For example, if we look at Figure 5, we will see that both the UK and the US face a similar 10y bond yield. In fact, both curves follow the same downward trend, and even have the same local maxima and minima, throughout the years examined. This reveals the perception of investors, that both these countries are considered safe investments and their economic prospects are positive.

[19] https://data.oecd.org/interest/long-term-interest-rates.htm

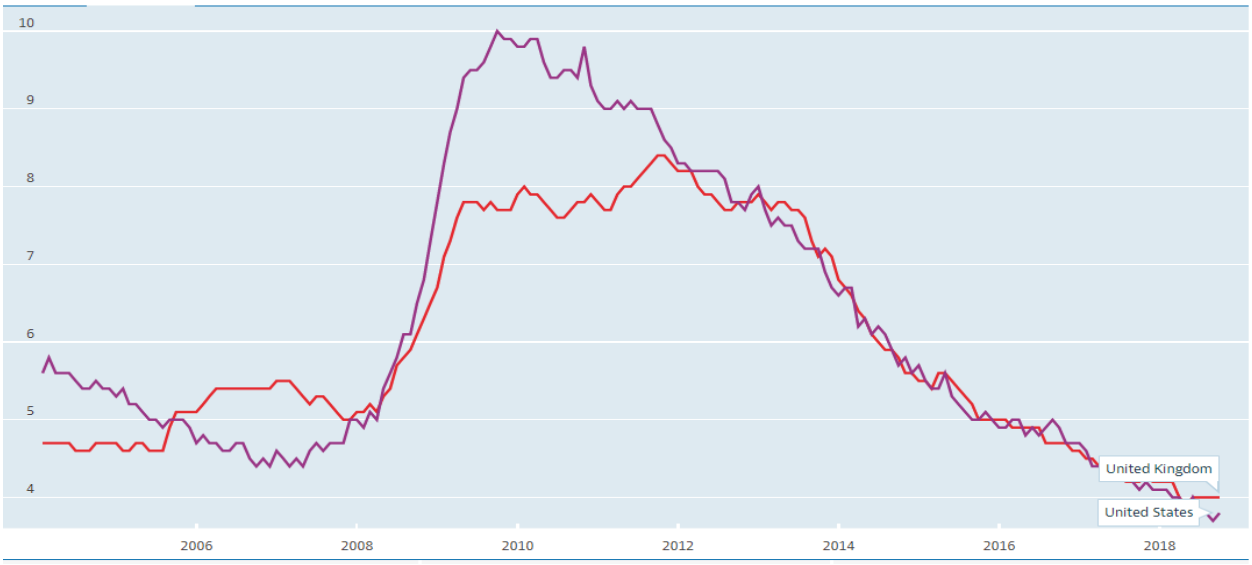


Figure 6: Comparison between the UK and US unemployment rate for the years 2004 – 2018 (Source: OECD[20])

The unemployment rate (Figure 6) is also very low in both countries and follows a similar pattern, at 4.9% for both of them in 2016, with the US being at a relatively better state after the implementations of legislative motions under the umbrella of the JOBS Act in 2011 - 2012, that was aimed at providing incentives to small and medium enterprises to raise more capital both as public and private entities, by decreasing the disclosure levels required by these companies in all of their activities. What is remarkable in this occasion is, that yet once more both economies show signs of a quick recovery from the financial crisis, as the unemployment rate drops at a rapid pace after 2010 for the US and after peaking in 2012 for the UK. Similar patterns can be revealed for other key macroeconomic variables, such as the inflation rate (see Figure 7 below), which was 0.7% for the UK and 2.3% for the US.

[20] https://data.oecd.org/unemp/unemployment-rate.htm

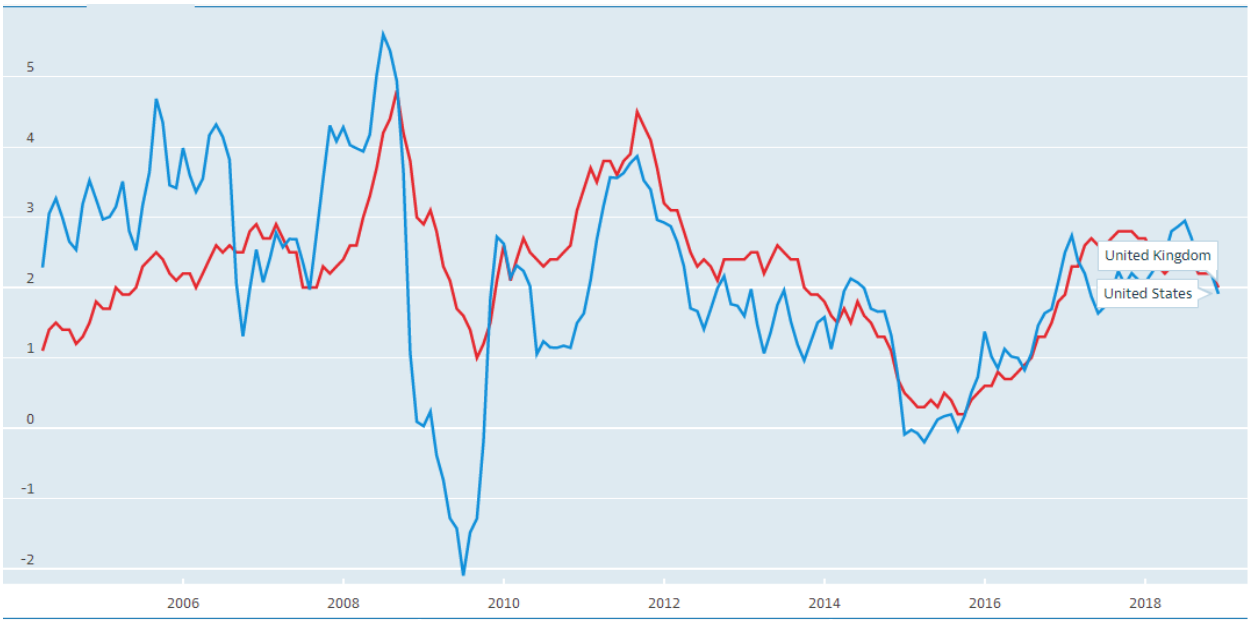


Figure 7: Comparison between the UK and US inflation rate for the years 2004 – 2018 (Source: OECD[21])

Another noteworthy characteristic of these economies is how they are structured, or to be more accurate which their major products are. Specifically, the UK is primarily an economy focused on services, and to a lesser extent on industry. Similarly, to that pattern, the US also is focused on services, while having industrial production as a secondary sector of its economy. Taking into consideration everything, mentioned so far for these countries, it suggests that they are very similar (in this thesis context, as in reality there are major size differences), as they are mature economies, which share many similarities, which can be used to draw contextual inferences.

Aside from the graphical representation of the various macroeconomic measures, another point that can be made is the similarities in how these economies are structured. To illustrate this, we can look at the Office for National Statistics (ONS) for the UK[22] and the Bureau of Economic Analysis (BEA) for the USA[23], where it is explained how each of the production sectors contribute to the GDP. For both the UK and the US the major contributor is the Service Sector, with a percentage close to 80%, followed by the Industry

[21] https://data.oecd.org/price/inflation-cpi.htm#indicator-chart
[22] https://www.ons.gov.uk/economy/grossdomesticproductgdp/bulletins/grossdomesticproductpreliminaryestimate/octtodec2015
[23] https://www.bea.gov/data/gdp/gross-domestic-product

Sector that contributes almost 20% and the rest is covered by Agriculture[24]. More in depth analysis reveals also that for both of these countries the GDP growth was similar for the year 2015 (2.4% in the UK and 2.9% for the U.S.), GDP could be mainly attributed to household consumption, the industries contributing to the GDP being similar, with the exception of the high-tech firms being more important contributors in the US, and public debt being in almost similar levels (87% of GDP for the UK and 81% for the US). All these similarities reveal that both of these countries are similarly structured in terms of how they produce wealth and have a similar dynamic in terms of growth.

All the above, are important, as they provide further support around the decision on which countries to focus on and from which we form the data sample. Another reason that spurred the interest in these countries is the availability of data. Focusing on developed countries with an established corporate reporting system (that dates back over 100 years) and both operate common law systems, as well as a similar legal framework in terms of contract law, suggests that we will be able to find accurate data to test the original hypotheses. This was a major consideration, especially at the beginning of the thesis, as there are many errors in companies' data (especially as the main focus is on private companies).

3.3.3 Market Selection

Having selected which countries to focus on, the next decision we have to make is the respective markets from which we will draw the sample. We were aided in that task by a series of papers. The most important of all of them is the study of Gerakos et al. (2013) which has already been discussed in the previous section. The paper focuses on the Alternative Investment Market of London (AIM) and compares the returns on the listed companies' stocks to those of the NASDAQ, OTCBB and Pink Sheets, from the US. Their focus is mainly on how these returns are affected by the different levels of oversight these markets have. These markets also represent different kinds of risk for their investors. Both NASDAQ and OTCBB represent the more traditional markets, in the sense that they are both regulated and monitored. Similarly, AIM is also

[24] A more condensed report for both countries with extensive macroeconomic data can be found in: https://www.cia.gov/library/publications/the-world-factbook/geos/us.html

regulated however it allows a higher degree of freedom in terms of reporting and regulation to its constituents. Finally, the Pink Sheets represent the highest risk investments, as they are not regulated and represent very small firms with a higher probability of default.

Following the previous study's example, this thesis will be focused primarily on the AIM and NASDAQ markets. What led to this decision is that, as explained in Gerakos et al. (2013) and Farag, Mallin, and Ow-Yong (2014), AIM is addressed to growth firms, namely younger less established firms, that need to raise capital but do not fulfill the requirements to enter a more traditional market (as there are minimum capital requirements), or do not want to engage in the more established markets' costly regulatory and disclosure practices. In this thesis we will be using the Mid Cap and below[25] of the NASDAQ, as we want the samples to be matched following the example of the study of Gerakos et al. (2013).

The choice of the AIM and the NASDAQ is further driven by a more primary rationale. As we have already discussed this thesis attempts to draw conclusions that are transferable from public companies to private ones. For the research results to have this property, we need to make sure that the sample used in the first part of the thesis, namely the public companies, will be as comparatively close as possible to the secondary sample, the private companies. As such we need to make sure that the public companies will be faced with as many restrictions as the private companies are. In that sense, a market with small and medium companies, which can be best described as growth companies, which operate in an environment of reduced regulation and disclosure, approximates the private companies, that also are essentially small to medium in size, operate under reduced disclosure and face liquidity issues. For that reason, the AIM is a good candidate to act as a proxy in the UK. For the NASDAQ, the disclosure part is not similar, however we can assume a relatively small size for the companies (for most of the parts that we focus on), as well as similar risk profiles for them in the USA.

Before moving on to the final part of the data section, namely the sample construction and the variable selection, it would be prudent to consider how the markets that are going to be used in the first of the two

[25] The NASDAQ below Mid Cap, also has Small Cap, Micro Cap and Nano Cap.

stages of the analysis operate and what their requirements for listings are. This will allow for a better understanding of the types of companies that will constitute the sample and will shed some light on the differences between the companies in the two indices.

Since its beginning in 1995, the AIM has been highly successful in its purpose, mainly due to the lessened disclosure requirements that are a typical characteristic of the more traditional senior public equity market. The companies listed on it are under the advisory supervision of what is called a NOMAD, which is an acronym for nominated advisor, who is responsible for the firm being truthful in its dealings with its investors and is an individual firm directly registered to the London Stock Exchange (LSE). These NOMADs are used as proxies for the LSE and oversee the companies' compliance with the rules and regulations of the exchange[26], with severe penalties imposed not only on the firms but on the advisors themselves in case of non-compliance. The NASDAQ, on the other hand requires their listed firms to meet the requirements of at least one of the markets subsections (global, national and capital market), which can be summarized as meeting specified minimum levels for the number of their publicly traded shares, total market value, stock price, and number of shareholders[27]. It can be easily observed from the above, that AIM is in general more flexible and therefore more attractive to smaller firms.

3.4 Sample Construction and Variable Selection

3.4.1 Sample

The sample for the first part of the thesis, will be constructed from panel data on companies listed in the AIM and NASDAQ markets over the period of 2004 to 2015. The sample for the secondary part of the thesis will be formed from variables created by applying the methodology and will be related to UK and US private enterprises' data for the same period. This time frame was chosen mainly for two reasons. The

[26] A complete list of the duties of all the market participants in the AIM can be found at: https://www.londonstockexchange.com/companies-and-advisors/aim/regulatory-landscape/regulatorylandscape.htm
[27] A complete list of the requirements is available at http://nasdaq.cchwallstreet.com/

first one is that 12 years of data are a sufficient source of variation for any researcher to draw inference from larger samples allow for more variation incorporation within the sample and thusly lead to less biased results, as those encapsulate all aspects of the relationship between the dependent and independent variables (Gujarati and Porter, 2009). The second reason is the financial crisis. As the sample is almost balanced around the year 2008, albeit a bit skewed to the years after the crisis, it will provide enough data to be able to observe the effects that not only the crisis had upon the discount rates but also the effect of the legislation that was passed in the crisis' aftermath. It is also preferable to allow the effects of external shock to fully develop throughout time, as there are always residual effects to be observed (Love, Preve, and Sarria-Allende, 2007).

To build the first sample we use several studies as guides. Following Gerakos et al. (2013) we choose the appropriate companies to match the AIM and NASDAQ sample. As in that study, we acquire a historical list of all the company listings in the AIM[28] for the period we are interested in, namely from the 1st of March in 2004 till the same day and month in 2016 (to include the full fiscal year extended into 2016). Out of the 15,260 listings we found over these years, we excluded a number of those for several reasons. Firstly, as is the norm in the accounting and finance literature (see the paper of Ellis, 2006, for example), we excluded all financial, REITS and closed-end funds. This can be attributed to the fact that financial related companies are subject to a higher degree of regulation, as well as different risk factors and valuation as compared to the rest of the market. Furthermore, we remove duplicate listings and firms that do not have adequate data recorded on Bloomberg and Capital IQ (missing variables or variables with non-verifiable data).

To match this initial sample we follow the methodology proposed by Gerakos et al. (2013) (we deviate from it as we do not match each company separately or exclude them if they have no match, however we create boundaries for the samples so as to have companies within the same capitalization limits in the samples) and match the AIM sample firms with firms from the NASDAQ, based on the total capitalization

[28] There are comprehensive lists of all the companies arranged by sector in the LSE's website: https://www.londonstockexchange.com/statistics/historic/aim/aim-statistics-archive-2015/dec-15.pdf

in their respective markets. We end up with 1,126 companies for the AIM. As we want to create a solid base for comparison, we follow a similar procedure, as the one described above, to construct the sample for the NASDAQ index, over the period of 1st of January in 2004 to the 31st of December of 2015. The final sample for this gives us a sample size of 3,846 companies in total. In the table below (Table 4), we present the distribution of companies in the AIM and NASDAQ sample.

	AIM	**NASDAQ**
Industry (*UK SIC*) / *(ICB)*	*No. of Companies*	*No. of Companies*
Oil and Gas (*0000*) */ (0001)*	147	247
Basic Materials (*1000*)	179	346
Basic Industrials (*2000*)	224	382
Consumer Goods (*3000*)	75	342
Health Care (*4000*)	116	1096
Consumer Services (*5000*)	157	437
Telecommunications (*6000*)	42	191
Utilities (*7000*)	37	223
Technology (*9000*)	149	582
Total Companies	***1,126***	***3,846***

Table 4: Number of Companies in the Sample based on the industry they operate in. The UK samples consists of companies from the AIM All Share Index, while the US sample consists of the Mid, Small, Nano and Micro-Cap Sections of the NASDAQ Index. The UK SIC codes and the ICB codes[29] for each industry group are in parentheses next to the name of the group.

The difference in the sample size for the two markets can be attributed to various factors. As we have already established AIM is addressed to growth firms[30] that do not want to go "public" in the strict sense of the term or do not have the capital required to do so. They want however to raise as much capital as possible to fund their operations, while not having to deal with increased disclosure requirements.

[29] The NASDAQ changed its classification codes on the 1st of January 2019, however we report the ICBs (Industry Classification Benchmarks) with the old codes, as these were used when we obtained the data originally in 2017. A conversion map between the new and old codes can be found at: https://www.ftserussell.com/financial-data/industry-classification-benchmark-icb

[30] There are also large firms in the AIM, they are however a small proportion of the sample. For example, in the year 2015 only 4 companies were above 1 billion GBP, while the larger part of the population resided between 2-250 million GBP. Source: https://www.londonstockexchange.com/statistics/historic/aim/aim-statistics-archive-2015/dec-15.pdf

As Gerakos et al. (2013) also explain the AIM market flourished and in 2006 it even raised more total capital than the LSE main market, as companies always look forward to increasing their capital, especially when this increase is associated with lower requirements to do so. This trend however discontinued after the financial crisis of 2008. NASDAQ listings on the other hand are companies that have adequate capital to dispense and there was an increase in the listings after the easing of the requirements provided by the newly passed legislation in the US, after the financial crisis of 2008 (see for example the study of Chaplinsky, Weiss, and Moon (2017)) that explains how the reduced costs induced by the JOBS Act, increased listings and trading activity in the US).

A final remark on the composition of the sample. The AIM portion of it seems to be more balanced. Specifically, the major contributors, namely the basic industrials (20%), materials (16%) and services (14%) account for 40% of the total AIM sample, however oil and gas (13%), healthcare (10%) and technology (13%) do not fall far behind from the first 3 sectors and in fact account for almost the other half of the sample. The NASDAQ portion on the other hand, seems to be more skewed towards the healthcare sector, which accounts for almost 30% of the sample, with most of the other sectors contributing between 5% and 10% each. The only notable exception is the technological sector that is almost at 15%.

3.4.2. Variable Selection

Since we have determined already which companies will be the focus of this study, the next step is to look into the specific variables that we will need to evaluate in order to answer the series of research questions presented in the previous section. As we have already established the literature thrives with factors that can be used as potential variables, this literature will be used to determine appropriate variables. The data drawn will be from numerous sources, such as Bloomberg, Capital IQ, the Bank of England, the US Treasury, OECD (for most of the macroeconomic data), the SEC and LSE websites. Moreover, we will utilize the Bureau van Djik M&A database, to create a unique dataset of private enterprises, which we will use to test the results from the methodology we will propose in the next section. We will also utilize the online

database of Aswath Damodaran[31]. In short, we will be using macroeconomic, legislative and company-specific variables. What follows is the complete list of the variables and their definitions, categorized between macroeconomic legislative and company related factors:

Macroeconomic

- *Consumer Confidence Index* is a score from an index published by various sources[32] an indicator of the expectations of citizens, on their country's economy. There are several institutions reporting the consumer sentiment, and it is arguably a good indicator on the current and predicted economic activity. One of the most prominent ones is the index created by the University of Michigan and the one created by the Conference Board. The criteria used in it include answers from random households on 50 different questions regarding the general economic conditions, with a specific focus on interest rates, inflation and job availability and how the citizens view the current and future conditions. A detailed description on the index and how it affects risk can be found in Fisher and Statman (2003).
- *GDP Growth* as found in Chordia and Shivakumar (2006), is the percentage change compared to the previous year, with GDP defined as the market value of all the goods and services produced in a country (data for the GDP Growth taken from the OECD's database).
- *Short-Term Interest Rate* is defined as the central-bank's (Bank of England and US Federal Reserve, in this case) determined lending interest rate, which is designed to provide liquidity to the country's market and is reflected in the one-year treasury bonds' yield. This has been identified as a common risk factor in Duffee (2006) and Koutmos and Philippatos (2007) (data for the short-term interest rate were downloaded from OECD's database).
- *Long Term Interest Rate* is defined as the bond yield of the ten-year government bond and reflects the expectations over a country's capability to meet its obligations towards its lenders, as a function of the

[31] The online database is on this website: http://people.stern.nyu.edu/adamodar/ and is a very good source for all kinds of data for private and public companies.
[32] Most of our macroeconomic data came from the OECD's website (https://data.oecd.org/) and for the CCI in particular the source can be found in https://data.oecd.org/leadind/consumer-confidence-index-cci.htm, however there are other reliable sources on this topic (see for example the Conference Board https://www.conference-board.org/).

various macroeconomic determinants that act as economic health indicators (Jiménez, Ongena, Peydró, and Saurina, 2014) (data for the long term interest rate were obtained from the OECD's database).

- *Yield Spread* is considered a measure of the country risk premium and is defined by Amira (2004) and Campbell and Yogo (2006), as the percent difference between Moody's triple A seasoned corporate bond yield and the one-month T-Bill (or other Treasury issues in the case of the UK). This variable was obtained from Bloomberg's bond database.
- *ICRG Political Risk Rating* is a cumulative score from several sources, including socio-economic factors, to estimate the political risk in a country that ranges from 0 to 100. Specifically, it looks into factors such as the perceived level of corruption in a country, government stability, the economic conditions (as those are expressed by the Confidence Index, Unemployment and other economic measures), and the ability of a country to attract investments among others. A comprehensive guide on the subject can be found in the study of Bekaert et al. (2016)[33].
- *Unemployment Rate* is the ratio of the people who are actively seeking employment but there is not enough supply to cover the demand, over a country's total workforce. It is one of the most referenced indicators of the economic situation of a country (Solow, 1980) (data on unemployment rate were downloaded from the OECD's online database).
- *Inflation* is a measure used in numerous studies (for example Duffee, 2006; Fama, 1981; Wachter, 2006), and is defined as the rate at which the prices of a predetermined basket of goods are changing (inflation data were obtained from the OECD's database).
- *Output Gap* is measured as the difference between actual and highest potential GDP, or to put it in simpler terms, whether an economy is utilizing all the available resources. This is a measure that has been relatively recently introduced in the risk premia literature by studies such as Cooper and Priestley (2009) (data on the Output Gap can be found on the OECD's website).

[33] Data for the ICRG Political Risk were downloaded from Harvard's database: https://dataverse.harvard.edu/dataset.xhtml?persistentId=hdl:1902.1/21446

- *Financial Crisis* is a binary variable that refers to the economic crisis that started in the US in 2008 and get the value of 1 if the company operated from 2008 and to the end of the sample, else 0. It is a variable well used in the literature, in studies such as Love et al. (2007).
- *Volatility* is measured as the standard deviation of the market index (AIM and NASDAQ respectively) over a specific period. In this case, we used the 90-day volatility as proposed by Mele (2007) (all the data for this variable have been obtained from Bloomberg).

Legislation

- *IAS 34*, this variable is specific to the AIM companies and is binary, with the value of 1 if the period under which the company is examined had this standard implemented, else 0. For binary variables that refer to standards or other legislation we use 1 as the value of the variable from the moment the legislation was implemented till the time it stopped being in effect. If that legislation is still ongoing, the variable takes the value of 1 from the moment it was implemented through the end of the sample (all years after the implementation of IAS 34 get the value of 1). This specific standard refers to the shift in the reporting requirement that took place in the AIM market after the adoption of the International Accounting Standard 34, which presented the companies with the opportunity to report either once every 6 months or once in a twelve-month period.
- *Companies Act 2006* was targeted to all UK companies, including AIM companies, and is binary, with the value of 1 if the period under which the company is examined, else 0. (As explained above the variable gets the value of 1 from 2006 till the end of the sample). This piece of legislation affected several aspects of a company, as it simplified company law, bettered shareholder rights and made reporting easier. This variable is also used in the study of Michaely and Roberts (2012).
- *IAS Framework Adoption*, is binary and specific to the UK market, with the value of 1 for the time period under which the company is examined, else 0, (as it is explained above) and refers to the adoption of the new IAS framework in 2005 by the AIM (Gramlich, Mayew, and McAnally, 2006).

- *JOBS Act* is a binary variable specific to the US, with the value of 1 if the company operated during 2012 till the end of the sample, else 0 (similarly to what was mentioned before), and refers to a series of legislative acts that started after the financial crisis and allowed firms to raise capital more easily, regardless of whether they were public or private (Chaplinsky et al., 2017).
- *FAS 123R* is a binary variable specific to the US, with the value of 1 if the period under which the company is examined had this law implemented, else 0. This legislative act, that was implemented in 2005 in the US, affected the way that financial options were calculated, and subsequently changed the executive compensation's structure, to a more performance-based approach (Hayes, Lemmon, and Qiu, 2012) and in this way affected the relationship between the management and the investors' interests.
- *Entry Amendments* is a binary variable specific to the US, with the value of 1 if the company in the sample operated during the period that these amendments were implemented, else 0, refers to a series of changes that affected companies entering the NASDAQ after 2005, that increased disclosure level requirements for the listed companies.

Company-related

- *Effective Tax Rate* is the tax expense expressed as a percentage of pre-tax income (Krüger, Landier, and Thesmar, 2015) and has been considered as one of the key factors in creating value for investors (Grinblatt and Liu, 2008) as it affects the discount rates.
- *Big 4 Auditors* is a binary variable, with the value of 1 when one of the big four auditing companies has been used by the company, else 0. Reputable auditors are important as they signal good quality in the financial statements (De Franco et al., 2011).
- *Intangible Assets* are measures of the value of non-physical assets, namely brand significance, patents and trademarks, goodwill etc., as those are reflected on the balance sheet and are in many cases highly regarded by analysts and appraisers, as they signify the potential for sales and growth (Gu and Wang, 2005). We use the natural logarithm of the value of the Intangible Assets.

- *Tobin's Q* is the centerpiece of modern asset pricing theory and has numerous supporters (e.g. Allayannis and Weston, 2001; Coles, Daniel, and Naveen, 2008; Gao and Zhang, 2015). It is defined as the total market value of a company over the book value of its assets, and it highlights the real value of a company compared to the replacement cost of its assets (as proxied by the book value). For our study we have used the natural logarithm of the variable.
- *Risk Management Practices* is a binary variable, with the value of 1 when the company engages in risk management practices (for example if the company uses derivatives for hedging as reported by Bloomberg), else 0, and is an important one to examine as such practices potentially reduce the company-specific risk (Vickery, 2008).
- *Assets* is the most commonly used variable in the accounting and finance literature, with several papers using them as a proxy for the value of a company (some examples would be Bebbington and Larrinaga-González, 2008; Fama and French, 2007; Richardson and Waegelein, 2003) and is defined as the natural logarithm of the total assets of a company.
- *EBITDA* defined by Platt and Platt (2002) as the natural logarithm of the sum of the operating income and the depreciation and amortization.
- *R-Square* defined by Damodaran (2002), as the percentage of variation of the returns explained by regressing stock returns on market returns. This measure is particularly useful in the explanation of the Total Beta.
- *ROE* defined by Wilcox (1984) as net income over the book value of shareholder's equity.
- *ROC* defined by Kiani, Chen, and Madjd-Sadjadi (2012) as the earnings before interest and taxes times 1 minus tax rate over the total value of debt and book value of equity minus the available cash.
- *Retention Ratio* measured as the percentage of earnings reinvested in the firm, is often viewed as good indicator for future growth but it signifies that dividend payments are proportionally less, which negatively impacts stockholders (Penman, 2010).

- *Depreciation* measured, also in Penman (2010), as the percentage loss of value for the company's assets over a finite period as reflected on a company's balance sheet.
- *Earnings Yield* defined by Durré and Giot (2007) as the annual earnings per share (EPS) over the market price of the share.
- *Debt / Equity* is defined as the total debt over the book value of the total equity of a firm and measures the ability of the company to meet its long term obligations (Korteweg, 2010).
- *Enterprise Value* is described in De Franco et al. (2011) as the natural logarithm of the sum of the market value of equity and debt less the available cash.
- *Insider Stock Percentage* is the number of company's shares that inside directors, hold (inside directors are those employed by the company) over the total shares of the company.
- *Board Composition* as defined by Bertoni, Meoli, and Vismara (2014) is the number of inside and outside directors on a company's board. In this case we use the number of outside directors, as a good number of external directors signifies the board independence and is a sign of good control over the company's management.
- *Compensation / Assets* is a ratio defined, as the name suggests, as the total compensation of the CEO over the company's total assets recorded on the balance sheet, and is one measure of management performance in relation to the company's performance (Brick, Palmon, and Wald, 2006).
- *No. of Insiders Holding Stock* is a variable that is defined as a percentage of insider holding stock over institutional stock holdings, that act as a governance characteristic similar to the other two noted previously, inspired by several studies in the corporate governance field and executive remuneration (see for example the study of Bae, Stulz, and Tan (2008), who find that corporate performance is positively associated to insiders' stock holdings).
- *Inventory / Sales*, is an activity ratio, defined in Kiani et al. (2012) as the inventory over revenues.

- *Net Operating Margin* defined in Davis and Peles (1993) as the net income of the company over its revenue. This is essentially a measure of the profit left in the company after all other expenses have been managed.
- *Dividend Growth* is the percentage rate increase or decrease of the dividends over a time period. This variable is present in all the dividend discount models that estimate the price of a stock (Chen, 2009).
- *FCFF* defined in Schatzberg and Weeks (2004) as the natural logarithm of the sum of the net income, non-cash charges and the after-tax interest, minus fixed capital investment and working capital investment during the period.
- *FCFE* defined by Rajan and Wulf (2006), among others as the natural logarithm of the net income minus the net capital expenses minus the changes in the net working capital plus any new debt minus the repayments of old debt, and is a measure of how much cash is available to be given to the shareholders.
- *Cap. Expenditure* is the natural logarithm of the sum of the funds used by a company that increase the value of its physical assets. This variable was referred in the paper of Jian and Lee (2011), among others.
- *WACC* is defined by Fernández (2011) as the sum of the product of the cost of equity times the percentage of financing that is comprised of equity at market value plus the product of the cost of debt at market value times the percentage of financing that is debt times 1 minus the tax rate. This variable has been the centerpiece of many studies in the literature since Modigliani and Miller (1963).
- *Cost of Equity* is the product of the CAPM (Mangena et al., 2016).
- *Cost of Debt* is calculated as the sum of the risk-free rate plus the company's spread times the after-tax rate and is used as a variable in numerous research studies (Fernandez, 2003).
- *Z-Score* is a variable that originally appeared in the paper of Altman (1968), where a multiple discriminant analysis gives a number that serves as a measure of the probability of the company's default, based on its assets and liabilities. It is defined in the paper of An and Chan (2008) on the credit ratings of private firms going public, by the following formula: Z-score = 6.56 x (working capital / total assets) + 3.26 x (retained

earnings/ total assets) + 6.72 x ((EBIT / total assets) +1.05 x (book value of total equity / book value of total liabilities)).

- *MPK* (Marginal Profit ratio) defined in the paper of Cooper and Priestley (2016), as the earnings per share over the total equity's market value adjusted for the average value of the industry the company operates in (subtracting the average of the industry the company operates in). According to the authors, this measure, along with the next two in this list, serve as risk factors that affect the returns in specific industries.
- *ROA* (Return on Assets) defined in the paper of Cooper and Priestley (2016), as the net income over the company's total asset, adjusted for the average value of the industry the company operates in (subtracting the average of the industry the company operates in).
- *IK* (Investment to Capital ratio) defined in the paper of Cooper and Priestley (2016), as the total invested capital over the market value of total equity deflated using the average value of the industry the company operates in as a deflator (similarly to the previous two variables).
- *Alpha* defined originally by Jensen (1969) as the annualized return on stock minus the risk free rate plus the Beta times the annualized return on the market. According to Cochrane (2011) this variable acts often as a measure of managerial performance.
- *Beta*, using the definition of Damodaran (2003), is estimated through regressing weekly or monthly depending on the database, with this study using the former so as to better capture the changes in the sensitivity of the stocks) returns of a stock against a specific index using two or five years of data.
- *Total Beta* is defined as the market Beta over the correlation between the market and the stock. It is a measure originally proposed by Damodaran (1999), as a means to estimate the cost of equity, aimed to those companies, whose investors cannot properly diversify their investments. It has gained considerable support especially by practitioners over the past few years (after Butler and Pinkerton, 2006), as we have seen in the literature review.

The list of variables is long, it is appropriate however, since it matches the wide spread of factors identified in the literature. The goal is to capture all issues covered in the empirical and theoretical literature by combining them so that we should be able to assess the relative impact of each of them in the valuation process. It will become clear that this was done by design as the first stage of the proposed methodology will be reducing these 48 variables to a much smaller focused statistically significant framework number that we will use to draw conclusions that we will attempt to carry over to the private companies' evaluation, that are part of the second sample. As yet, we do not know which variables will emerge after the size reduction, we will discuss how we constructed the sample of the private companies after we have the results of the initial Principal Component Analysis (PCA). To understand how the final sample was constructed (in conjunction with the initial analysis we did in the beginning of the section), we also provide with the following table (Table 5):

Sample Construction (Obs.)				
	UK (Obs.)	**No. of Companies**	**US (Obs.)**	**No. of Companies**
Initial Sample	421764	15260	1515905	35497
Reduced Sample	92393	1126	132557	3846
Winsorized Sample	87590	1126	124896	3846
Final Sample	**87590**	1126	**124896**	3846

Table 5: This table reports the initial number of observations included in the sample as those were drawn from the data sources for all the variables, and the final number of observations that were used for the analysis part. We have also included the number of companies in the initial and the final sample. The process we followed was to normalize the data, log-transform the variables that required it and winsorized them. The first row shows the number of observations / companies, as those were drawn from our data sources. The second shows the number of observations / companies, remaining after we have applied our restrictions (described in detail in the preceding section) and the last row is the final sample after we have performed the aforementioned processes.

To reach the final number of observations on our sample (UK: 87590 and US: 124896), we have also taken some extra steps to ascertain, that our sample will be suitable for the process that will follow. The first one was to normalize the data. This is a requirement for PCA, as the methodology relies on the correlation or the covariance between variables. This suggests that we had to make sure that our data would be normally

distributed. We have also used log transformations on several of the variables, mainly those associated with Assets (which is our proxy for size and our means of matching companies at a later stage), whether those be tangible and intangible assets, or EBITDA, or Tobin's Q, to name a few (for a more complete overview on the logarithm transformations please look at the "measurement unit" column of the descriptive statistics tables.

Another step we took was to winsorizing the data. Specifically, after we have made certain of the normality distribution of them, we have eliminated the data at the 1% and 99% level, to make sure that our sample is free of outliers. The end result was the number of observations mentioned in Table 5. One might notice at that table that the data points were eliminated but the number of companies remained the same. This can be attributed to the longitudinal nature of our data, which ensures that removing specific observations does not mean we have to remove the entire time series (or the entire company the observations are part of).

We also include at this point (with the four tables that follow) detailed descriptive statistics, initially for the full sample (Tables 6 and 7) and also for further robustness descriptive statistics for the sample split between before and after the financial crisis, for the years 2004 till 2008 (Tables 8 and 9) and 2009 till 2016 (Tables 10 and 11). This is done also for another reason. It will help us get a first glimpse at any potential effect an external shock might have on the original variables. Indeed the financial crisis seems to have an impact on several fundamental variables (for instance EBITDA and the Debt to Equity ratio).

Tables 5 and 6 present the descriptive statistics for the variables on both the AIM and the NASDAQ for the full sample.

Descriptive Statistics												
	UK						US					
Variable	Mean	Median	Std. Dev.	High	Low		Mean	Median	Std. Dev.	High	Low	Measurement Unit
Consumer Confidence Index	99,93	100,51	1,57	101,89	96,69		99,28	99,48	1,32	101,32	96,71	*Scalar*
GDP Growth	0,01	0,02	0,02	0,03	-0,04		1,88	2,37	1,67	3,78	-2,77	*Percentage*
Short - Term Interest Rate	3,66	4,59	2,15	6,30	0,48		1,87	0,61	1,97	5,42	0,11	*Percentage*
Long - Term Interest Rate	4,09	4,44	0,97	5,43	1,65		3,38	3,45	1,06	5,08	1,52	*Percentage*
Yield Spread	0,43	-0,19	1,41	3,49	-1,61		1,51	1,74	1,29	3,64	-1,27	*Percentage*
ICRG Political Risk Rating	5,20	5,23	0,13	5,40	4,83		5,12	5,13	0,09	5,24	4,98	*Scalar*
Unemployment Rate	6,00	0,05	0,01	0,08	0,05		6,57	5,80	1,78	9,60	4,60	*Percentage*
Inflation	2,39	2,32	0,88	4,48	0,05		2,23	2,67	1,23	3,83	-0,35	*Percentage*
Output Gap	0,18	1,09	2,62	2,91	-4,46		-0,78	-1,29	2,80	3,05	-4,56	*Percentage*
Financial Crisis	0,26	0,00	0,44	1,00	0,00		0,72	1,00	0,45	1,00	0,00	*Binary*
Volatility	55,57	45,26	41,64	105,40	0,00		53,97	50,01	32,21	107,40	1,58	*Scalar*
UK IAS 34 / 2004 US JOBS Act	0,08	0,00	0,26	1,00	0,00		0,30	0,00	0,46	1,00	0,00	*Binary*
UK Companies Act 2006 / US FAS 123R	0,74	1,00	0,44	1,00	0,00		0,81	1,00	0,39	1,00	0,00	*Binary*
UK IAS Framework Adoption / US Entry Amendments	0,87	1,00	0,34	1,00	0,00		0,53	1,00	0,50	1,00	0,00	*Binary*
Effective Tax Rate	21,83	26,07	8,65	29,91	19,14		38,27	38,29	2,19	40,00	0,00	*Percentage*
Big 4 Auditors	0,97	1,00	0,18	1,00	0,00		0,59	1,00	0,49	1,00	0,00	*Binary*
Intangible Assets	8,27	8,05	1,97	10,08	0,00		2,17	3,21	2,32	7,13	0,00	*Logarithm/Currency*
Tobin's Q	2,47	1,69	2,61	3,11	1,07		3,56	3,73	1,48	9,34	1,66	*Scalar/Logarithm*
Risk management Practices	0,40	1,00	0,49	1,00	0,00		0,89	1,00	0,31	1,00	0,00	*Binary*
Assets	3,47	3,82	1,39	9,12	0,48		5,78	6,71	1,81	10,21	2,28	*Logarithm/Currency*
EBITDA	2,96	1,78	2,35	5,74	-4,03		3,04	2,86	2,41	6,25	-4,84	*Logarithm/Currency*
R - Square	14,22	6,25	0,18	21,09	2,05		29,27	25,05	0,25	100,00	0,00	*Percentage*
ROE	10,12	9,56	6,98	49,62	0,67		13,67	9,72	8,48	49,84	0,35	*Ratio / Percentage*
ROC	14,02	12,67	7,05	49,91	1,76		10,31	7,23	3,54	49,89	0,13	*Ratio / Percentage*
Retention Ratio	0,87	1,00	0,12	1,00	0,08		0,86	0,61	0,16	1,00	0,22	*Ratio*

Table 6: This table presents descriptive statistics on the overall sample. The measurement unit specifies how each variable is denoted. Furthermore, we present the natural logarithm of intangible and tangible assets, since we used this form for the PCA and Panel Regressions, and the scaled numbers are easier to interpret. Variables noted as "Currency", are calculated in millions. For the negative prices in the natural logarithms, we used the logarithmic transformation suggested by Elnathan et al. (2010), that allows for variables to scale while retaining the initial information on whether the variable has a positive or a negative price (the full analysis on this technique is included in the following section where we discuss the construction of our dependent variable). Finally, for the legislative variables, we denote whether they refer to the UK or the US, by including the words UK or US in front of the respective legislative act.

Our descriptive statistics do not reveal anything out of the ordinary, or to be more accurate, something we did not expect given the differences in the Indices our sample companies are listed in. Starting with the macroeconomic variables, they further attest to the sentiment expressed in the beginning of this section. Both countries are strong economies (although the US seems to be averaging higher GDP growth), highly developed with increased potential for growth and low political risk (the political score is equally close at 5.20 and 5.12 for the UK and the US respectively considering that the maximum of this measure's scale is 100). Low unemployment and inflation rates further attest to that notion. The output gap's descriptive statistics are also telling of the situation, as the UK's economy seems to almost fully use their resources, while the US shows potential to perform even better in terms of resource allocation and utilization, as indicated by the negative value of -0.78. Their markets are also characterized by similar high volatility in the 90-day window, which signals the potential for investors to realize high returns. We also need to mention at this point that we used the natural logarithm for variables with high values (for instance the

assets) following the related literature (see for instance the papers like those of Elnathan et al., 2010, or Minnis, 2011), in an attempt to scale the variables and make them more suitable for the purposes of regression analysis (also making the variables more easily interpreted in the process).

Descriptive Statistics (Continued)												
	UK						US					
Variable	**Mean**	**Median**	**Std. Dev.**	**High**	**Low**		**Mean**	**Median**	**Std. Dev.**	**High**	**Low**	**Measurement Unit**
Depreciation	3,29	2,94	6,14	57,22	0,00		12,47	13,02	21,36	261,23	0,00	*Currency*
Earnings Yield	10,95	6,51	11,93	81,76	-12,43		-1,83	3,34	13,93	91,16	-61,24	*Percentage*
Debt / Equity	25,68	13,81	23,15	99,05	1,91		22,08	16,93	27,87	97,92	0,00	*Ratio/Scalar*
Enterprise Value	3,93	4,11	3,66	4,85	3,56		4,65	4,40	3,79	6,11	3,37	*Currency/Logarithm*
Insider Stock Percentage	20.2	2,02	4,64	100,00	0,00		53,90	80,45	36,47	100,00	0,00	*Percentage*
Board Composition	0,77	1,87	1,37	10,00	0,00		7,23	7,00	2,01	13,00	0,00	*Scalar*
Compensation / Assets	0,16	6,12	2,47	21,79	0,00		5,28	1,62	8,32	42,99	0,00	*Ratio/Scalar*
No. Insiders Holding Stock	1,28	1,00	0,76	10,00	0,00		1,74	0,00	2,65	16,00	0,00	*Scalar*
Inventory / Sales	37,74	66,69	30,79	98,47	0,00		9,62	14,61	10,18	116,01	0,00	*Ratio/Scalar*
Net Oper. Margin	0,18	3,41	0,71	4,49	-2,88		7,28	8,34	15,37	63,14	-26,57	*Percentage*
Dividend Growth	0,03	0,00	0,11	1,00	0,00		21,36	23,42	10,09	91,35	7,42	*Percentage*
FCFF	4,83	3,45	14,35	136,52	-90,51		71,25	83,47	28,12	489,52	-161,37	*Currency*
FCFE	-12,34	-11,93	28,37	214,26	-176,45		26,73	37,16	38,34	525,32	-186,43	*Currency*
Cap. Expenditure	-7,48	-1,19	26,37	0,00	-385,77		-11,39	-5,71	33,24	0,00	-511,43	*Currency*
WACC	7,46	7,36	3,97	21,76	0,09		9,01	8,99	3,55	23,49	0,02	*Percentage*
Cost of Equity	7,84	7,83	4,52	29,16	0,07		10,44	10,37	3,48	18,46	0,03	*Percentage*
Cost of Debt	2,79	2,68	2,33	14,68	0,02		2,50	2,49	2,15	18,88	0,11	*Percentage*
Z - Score	2,30	2,71	2,23	5,98	1,80		2,90	3,06	0,27	6,72	2,06	*Scalar*
MPK	1,28	0,84	0,27	4,33	0,21		2,45	1,92	0,59	5,25	0,44	*Ratio*
ROA	0,68	0,69	0,43	1,92	0,08		0,94	0,77	1,21	2,67	0,17	*Ratio / Percentage*
IK	7,92	7,96	4,56	13,07	2,01		6,23	7,44	4,39	9,73	2,99	*Ratio*
Alpha	-0,37	-0,68	6,53	26,63	-11,48		0,71	0,85	7,54	61,77	-49,92	*Scalar*
Beta	1,89	1,90	0,92	9,85	0,16		0,82	1,07	0,10	4,37	0,09	*Scalar*
Total Beta	4,95	5,07	2,37	8,62	3,06		1,02	5,22	6,56	8,34	0,43	*Scalar*
N = 87590							**N = 124896**					

Table 7: This table presents descriptive statistics on the overall sample

At the company level of variables there are some points that can be highlighted. Firstly, the AIM companies seem more prone to hire one of the Big 4 Auditors, than the companies in the NASDAQ, as is exhibited by the almost perfect score of 0.97 for the AIM as opposed to 0.59 in the NASDAQ. This is consistent with the related research (see for example the paper of De Franco et al. 2011), that explains the importance of auditing as a signal to potential investors, especially in growth firms, such as those in the AIM. Moreover, AIM companies' higher intangible assets mean (8.27), is another indication of the idea described above. NASDAQ companies, as more mature and operating in a more regulated environment also engage more frequently in risk management practices (the average is at 0.89).

Similarly, there are other indicators of the uniqueness of the AIM as compared to the NASDAQ. For example, Beta and Total Beta, which are measures of the riskiness of the company compared to the overall riskiness of the market, are significantly higher in the AIM sample and so is Altman's Z. To expand further on this, Altman's Z is a number with the lower limit of 1.8 and the upper limit at 3.2, with companies that score near or at 1.8 being more probable to go bankrupt, while the companies that score near or at 3.2 being able to meet their obligations (Saunders, 1999). In this sample the AIM companies are closer to 1.8 (2.3) which is also a sign of the higher risk associated with them.

Descriptive Statistics before Crisis (2004 - 2008)

	UK					US					
Variable	**Mean**	**Median**	**Std. Dev.**	**High**	**Low**	**Mean**	**Median**	**Std. Dev.**	**High**	**Low**	**Measurement Unit**
Consumer Confidence Index	99,93	100,51	1,57	101,89	96,69	99,28	99,48	1,32	101,32	96,71	*Scalar*
GDP Growth	0,01	0,02	0,02	0,03	-0,04	1,88	2,37	1,67	3,78	-2,77	*Percentage*
Short - Term Interest Rate	3,66	4,59	2,15	6,30	0,48	1,87	0,61	1,97	5,42	0,11	*Percentage*
Long - Term Interest Rate	4,09	4,44	0,97	5,43	1,65	3,38	3,45	1,06	5,08	1,52	*Percentage*
Yield Spread	0,43	-0,19	1,41	3,49	-1,61	1,51	1,74	1,29	3,64	-1,27	*Percentage*
ICRG Political Risk Rating	5,20	5,23	0,13	5,40	4,83	5,12	5,13	0,09	5,24	4,98	*Scalar*
Unemployment Rate	6,00	0,05	0,01	0,08	0,05	6,57	5,80	1,78	9,60	4,60	*Percentage*
Inflation	2,39	2,32	0,88	4,48	0,05	2,23	2,67	1,23	3,83	-0,35	*Percentage*
Output Gap	0,18	1,09	2,62	2,91	-4,46	-0,78	-1,29	2,80	3,05	-4,56	*Percentage*
Financial Crisis	0,26	0,00	0,44	1,00	0,00	0,72	1,00	0,45	1,00	0,00	*Binary*
Volatility	55,57	45,26	41,64	105,40	0,00	53,97	50,01	32,21	107,40	1,58	*Scalar*
UK IAS 34 / 2004 US JOBS Act	0,08	0,00	0,26	1,00	0,00	0,30	0,00	0,46	1,00	0,00	*Binary*
UK Companies Act 2006 / US FAS 123R	0,74	1,00	0,44	1,00	0,00	0,81	1,00	0,39	1,00	0,00	*Binary*
UK IAS Framework Adoption / US Entry Amendments	0,87	1,00	0,34	1,00	0,00	0,53	1,00	0,50	1,00	0,00	*Binary*
Effective Tax Rate	21,83	26,07	8,65	29,91	19,14	38,27	38,29	2,19	40,00	0,00	*Percentage*
Big 4 Auditors	0,97	1,00	0,18	1,00	0,00	0,59	1,00	0,49	1,00	0,00	*Binary*
Intangible Assets	8,24	8,53	1,36	10,08	0,00	3,43	3,88	2,32	7,13	0,00	*Logarithm/Currency*
Tobin's Q	2,40	1,41	2,14	3,11	1,17	2,16	1,48	3,51	9,34	1,93	*Scalar / Logarithm*
Risk management Practices	0,40	1,00	0,49	1,00	0,00	0,89	1,00	0,31	1,00	0,00	*Binary*
Assets	3,60	3,96	1,47	9,12	0,48	5,91	6,83	2,74	10,21	2,28	*Logarithm/Currency*
EBITDA	2,34	1,63	1,19	5,74	-4,03	3,05	2,71	1,24	6,25	-4,84	*Currency/ Logarithm*
R - Square	14,22	6,25	0,18	21,09	2,05	29,27	25,05	0,25	100,00	0,00	*Percentage*
ROE	8,76	6,37	7,06	49,62	1,47	13,64	10,68	9,71	49,84	0,52	*Ratio / Percentage*
ROC	13,70	14,60	11,27	49,91	1,76	11,04	8,63	6,57	49,89	0,13	*Ratio / Percentage*
Retention Ratio	0,87	1,00	0,12	1,00	0,08	0,86	0,61	0,16	1,00	0,22	*Ratio*

Table 8 : This table presents the descriptive statistics for the first part of the sample, up to the financial crisis, namely from 2004 till 2008.

The maturity of the NASDAQ companies is also expressed in their governance variables, since the mean for both board composition, which measures board independence, and how closely the performance of management is rewarded with higher compensation (show by the compensation / assets variable) is higher than their UK counterpart. Furthermore, NASDAQ companies seem to have higher growth on their dividends on average than the AIM (21.36 for the NASDAQ to 0.03 for the AIM sample), which is to be expected given their relative maturity in comparison with AIM companies. Another characteristic that should be discussed is the positive for the UK (10.95) and negative for the US (-1.83), mean earnings yield

value. These signs can be explained if we refer to the difference in the accounting standards in the UK and the US, which can be responsible for the cause of such a variance.

Descriptive Statistics (Continued) before Crisis (2004 - 2008)												
	UK						US					
Variable	**Mean**	**Median**	**Std. Dev.**	**High**	**Low**		**Mean**	**Median**	**Std. Dev.**	**High**	**Low**	**Measurement Unit**
Depreciation	21,86	20,37	8,58	57,22	0,00		32,86	31,72	24,01	261,23	0,00	*Currency*
Earnings Yield	12,48	11,37	10,06	81,76	-12,43		4,15	5,06	13,94	91,16	-61,24	*Percentage*
Debt / Equity	25,68	13,81	23,15	99,05	1,91		22,08	16,93	27,87	97,92	0,00	*Ratio/Scalar*
Enterprise Value	4,01	3,98	0,86	4,85	3,56		4,42	4,38	0,91	6,11	3,81	*Currency/Logarithm*
Insider Stock Percentage	20.2	2,02	4,64	100,00	0,00		53,90	80,45	36,47	100,00	0,00	*Percentage*
Board Composition	0,77	1,87	1,37	10,00	0,00		7,23	7,00	2,01	13,00	0,00	*Scalar*
Compensation / Assets	0,16	6,12	2,47	21,79	0,00		5,28	1,62	8,32	42,99	0,00	*Ratio/Scalar*
No. Insiders Holding Stock	1,28	1,00	0,76	10,00	0,00		1,74	0,00	2,65	16,00	0,00	*Scalar*
Inventory / Sales	37,74	66,69	30,79	98,47	0,00		9,62	14,61	10,18	116,01	0,00	*Ratio/Scalar*
Net Oper. Margin	3,27	3,32	0,58	4,49	-1,37		7,21	8,12	11,17	63,14	-22,93	*Percentage*
Dividend Growth	0,03	0,00	0,11	1,00	0,00		21,36	23,42	10,09	91,35	7,42	*Percentage*
FCFF	10,58	9,81	9,27	136,52	-83,21		85,79	88,94	19,37	489,52	-137,45	*Currency*
FCFE	17,38	8,28	21,12	214,26	-160,37		62,17	51,34	30,07	525,32	-162,37	*Currency*
Cap. Expenditure	-60,29	-57,14	12,36	0,00	-381,07		-81,46	-75,36	31,27	0,00	-498,04	*Currency*
WACC	7,46	7,36	3,97	21,76	0,09		9,01	8,99	3,55	23,49	0,02	*Percentage*
Cost of Equity	7,84	7,83	4,52	29,16	0,07		10,44	10,37	3,48	18,46	0,03	*Percentage*
Cost of Debt	2,79	2,68	2,33	14,68	0,02		2,50	2,49	2,15	18,88	0,11	*Percentage*
Z - Score	2,30	2,71	2,23	5,98	1,80		2,90	3,06	0,27	6,72	2,06	*Scalar*
MPK	1,28	0,84	0,27	4,33	0,21		2,45	1,92	0,59	5,25	0,44	*Ratio*
ROA	0,68	0,69	0,43	1,92	0,08		0,94	0,77	1,21	2,67	0,17	*Ratio / Percentage*
IK	7,92	7,96	4,56	13,07	2,01		6,23	7,44	4,39	9,73	2,99	*Ratio*
Alpha	-0,37	-0,68	6,53	26,63	-11,48		0,71	0,85	7,54	61,77	-49,92	*Scalar*
Beta	1,89	1,90	0,92	9,85	0,16		0,82	1,07	0,10	4,37	0,09	*Scalar*
Total Beta	4,95	5,07	2,37	8,62	3,06		1,02	5,22	6,56	8,34	0,43	*Scalar*
N = 87590							**N = 124896**					

Table 9: This table presents the descriptive statistics for the first part of the sample up to the financial crisis, namely from 2004 till 2008.

A final point we feel the need to make is with regards to the negative values on the FCFE for the AIM companies (-12.34 mean). As our AIM -listed companies, are primarily growth companies, they have an increased need for reinvesting their earnings towards projects that can fuel growth, and subsequently result in negative cash flows to equity. These results are consistent with those presented in the research study of Brush et al. (2000).

Descriptive Statistics after Crisis (2009-2016)												
	UK						US					
Variable	Mean	Median	Std. Dev.	High	Low		Mean	Median	Std. Dev.	High	Low	Measurement Unit
Consumer Confidence Index	98,16	99,98	1,62	100,78	95,47		99,01	99,35	1,37	101,11	96,13	*Scalar*
GDP Growth	0,01	0,02	0,02	0,03	-0,04		1,75	2,35	1,59	0,81	-2,95	*Percentage*
Short - Term Interest Rate	3,75	4,63	2,09	7,02	0,51		1,85	0,58	1,89	5,67	0,13	*Percentage*
Long - Term Interest Rate	4,05	4,52	0,85	5,72	1,63		3,12	3,51	1,09	5,34	1,29	*Percentage*
Yield Spread	0,52	-0,17	1,45	3,78	-1,75		1,62	1,81	1,33	4,52	-1,32	*Percentage*
ICRG Political Risk Rating	5,22	5,42	0,12	6,32	4,95		5,21	5,18	0,11	5,75	5,01	*Scalar*
Unemployment Rate	6,20	0,07	0,02	0,12	0,06		6,61	5,84	1,75	9,89	4,76	*Percentage*
Inflation	2,94	2,64	0,91	4,87	0,07		2,54	2,71	1,12	4,23	-0,25	*Percentage*
Output Gap	0,21	1,14	2,84	3,25	-4,67		-0,58	-1,34	2,95	3,98	-4,36	*Percentage*
Financial Crisis	0,34	0,00	0,47	1,00	0,00		0,81	1,00	0,52	1,00	0,00	*Binary*
Volatility	56,75	47,68	43,75	108,56	0,00		55,48	51,23	31,25	109,45	1,67	*Scalar*
UK IAS 34 / 2004 US JOBS Act	0,10	0,00	0,27	1,00	0,00		0,32	0,00	0,49	1,00	0,00	*Binary*
UK Companies Act 2006 / US FAS 123R	0,81	1,00	0,45	1,00	0,00		0,85	1,00	0,42	1,00	0,00	*Binary*
UK IAS Framework Adoption / US Entry Amendments	0,91	1,00	0,41	1,00	0,00		0,59	1,00	0,51	1,00	0,00	*Binary*
Effective Tax Rate	23,48	28,94	9,12	32,45	21,10		39,27	39,45	2,32	42,00	0,00	*Percentage*
Big 4 Auditors	0,98	1,00	0,21	1,00	0,00		0,61	1,00	0,52	1,00	0,00	*Binary*
Intangible Assets	8,01	7,81	1,13	9,83	0,00		4,21	3,98	2,67	6,92	0,00	*Logarithm/Currency*
Tobin's Q	2,39	1,44	2,06	2,10	1,07		2,48	1,86	1,36	7,01	1,66	*Scalar / Logarithm*
Risk management Practices	0,45	1,00	0,51	1,00	0,00		0,92	1,00	0,35	1,00	0,00	*Binary*
Assets	3,36	3,08	1,67	8,73	0,91		5,52	6,37	1,94	6,51	2,39	*Logarithm/Currency*
EBITDA	2,72	1,75	2,93	5,66	-3,74		2,98	2,81	2,92	6,08	-4,60	*Currency / Logarithm*
R - Square	14,39	7,59	0,23	24,59	2,17		31,45	27,98	0,37	97,43	4,57	*Percentage*
ROE	7,45	3,47	6,11	28,36	0,67		9,34	8,12	8,56	42,13	0,35	*Ratio / Percentage*
ROC	11,23	10,03	8,01	37,68	1,76		8,97	6,53	1,14	34,63	0,14	*Ratio / Percentage*
Retention Ratio	0,89	1,00	0,13	1,00	0,09		0,88	0,63	0,17	1,00	0,24	*Ratio*

Table 10: This table presents the descriptive statistics for the second part of the sample from the period after the financial crisis, namely from 2009 till 2016.

Following our descriptive statistics and analysis, we have also included the correlation matrices (Tables 12 and 13) for both samples. The correlation between the variables, is an important characteristic, as the main methodology is based on it.

Descriptive Statistics (Continued) after Crisis (2009 - 2016)												
	UK						US					
Variable	**Mean**	**Median**	**Std. Dev.**	**High**	**Low**		**Mean**	**Median**	**Std. Dev.**	**High**	**Low**	**Measurement Unit**
Depreciation	1,49	0,91	9,87	56,03	0,00		10,49	13,48	24,34	250,79	0,00	*Currency*
Earnings Yield	9,87	6,53	9,37	80,34	-11,27		3,67	4,84	12,57	79,64	-55,47	*Percentage*
Debt / Equity	26,72	14,31	24,67	98,12	2,03		24,57	19,86	28,12	96,48	0,00	*Ratio/Scalar*
Enterprise Value	3,84	3,73	0,74	4,84	3,27		4,13	4,01	1,92	6,09	3,37	*Currency/Logarithm*
Insider Stock Percentage	21,20	2,31	4,78	100,00	0,00		54,65	81,67	37,78	100,00	0,00	*Percentage*
Board Composition	0,78	1,95	1,42	12,00	0,00		7,65	7,50	2,36	14,00	0,00	*Scalar*
Compensation / Assets	0,14	6,39	2,78	21,14	0,00		5,96	1,74	8,36	43,65	0,00	*Ratio/Scalar*
No. Insiders Holding Stock	1,31	1,01	0,78	11,00	0,00		1,78	0,30	2,74	15,00	0,00	*Scalar*
Inventory / Sales	38,69	67,58	31,45	99,14	0,00		10,34	15,36	11,21	118,23	0,00	*Ratio/Scalar*
Net Oper. Margin	2,99	3,17	0,78	4,38	-2,88		6,98	7,48	19,35	58,19	-26,57	*Percentage*
Dividend Growth	0,28	0,00	0,12	1,00	0,00		22,45	24,69	11,52	92,14	7,42	*Percentage*
FCFF	5,49	3,25	11,07	125,63	-90,51		75,39	85,46	22,19	475,25	-161,37	*Currency*
FCFE	15,27	11,41	34,25	209,04	-176,45		17,23	49,35	47,20	514,17	-186,43	*Currency*
Cap. Expenditure	-50,73	-45,27	6,98	0,00	-385,77		-70,08	-62,14	30,19	0,00	-511,43	*Currency*
WACC	8,36	8,21	3,75	25,39	0,85		7,98	8,51	3,67	25,49	0,02	*Percentage*
Cost of Equity	8,63	7,98	4,69	34,19	0,13		12,45	11,73	4,36	19,36	0,03	*Percentage*
Cost of Debt	3,01	2,96	2,45	17,46	0,11		3,90	3,23	2,98	19,23	0,11	*Percentage*
Z - Score	2,41	2,97	2,49	6,14	2,31		3,01	3,75	0,50	7,58	2,55	*Scalar*
MPK	1,31	0,94	0,31	5,12	0,31		2,59	2,08	0,64	6,39	0,39	*Ratio*
ROA	0,57	0,61	0,39	1,85	0,07		0,64	0,69	1,22	3,01	0,15	*Ratio / Percentage*
IK	8,01	8,31	4,67	14,25	2,25		6,54	8,12	4,51	10,12	3,22	*Ratio*
Alpha	-0,36	-0,69	6,78	27,31	-12,10		0,78	0,91	8,36	65,48	-50,12	*Scalar*
Beta	1,88	1,89	0,93	9,76	0,17		0,85	1,12	0,12	4,36	0,11	*Scalar*
Total Beta	5,10	5,19	2,41	9,36	4,23		1,22	5,84	6,71	8,59	0,42	*Scalar*
N = 87590							**N = 124896**					

Table 11: This table presents the descriptive statistics for the second part of the sample from the period after the financial crisis, namely from 2009 till 2016.

Variables	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
1 *Consumer Confidence Index*	**1.00**																																																
2 *GDP Growth*	0.35	**1.00**																																																
3 *Short - Term Interest Rate*	-0.10	0.30	**1.00**																																														
4 *Long - Term Interest Rate*	-0.07	0.12	0.85	**1.00**																																													
5 *Yield Spread*	0.11	-0.38	-0.73	-0.50	**1.00**																																												
6 *ICRG Political Risk Rating*	0.62	0.62	0.33	0.30	0.40	**1.00**																																											
7 *Unemployment Rate*	-0.29	-0.55	-0.78	-0.60	0.77	-0.76	**1.00**																																										
8 *Inflation*	-0.86	-0.23	0.09	0.12	-0.05	-0.72	0.41	**1.00**																																									
9 *Output Gap*	0.19	0.65	0.86	0.63	-0.87	0.61	-0.90	-0.18	**1.00**																																								
10 *Financial Crisis*	-0.01	0.03	-0.83	-0.79	0.72	-0.25	0.65	0.02	-0.62	**1.00**																																							
11 *Volatility*	-0.07	-0.22	-0.14	-0.12	0.13	-0.14	0.16	0.03	-0.19	0.04	**1.00**																																							
12 *UK IAS 34*	0.33	0.18	-0.41	-0.55	0.24	0.36	-0.06	-0.54	-0.07	0.47	0.03	**1.00**																																							
13 *UK Companies Act 2006*	-0.37	-0.40	-0.28	-0.29	0.23	-0.52	0.56	0.35	-0.36	0.35	0.11	0.17	**1.00**																																							
14 *UK IAS Framework Adoption*	-0.30	-0.24	-0.20	-0.29	0.10	-0.41	0.42	0.35	-0.18	0.23	0.08	0.11	0.65	**1.00**																																						
15 *Effective Tax Rate*	-0.03	-0.08	0.02	0.02	-0.01	0.02	-0.01	0.05	0.01	-0.03	-0.01	-0.01	0.07	0.01	**1.00**																																					
16 *Big 4 Auditors*	-0.01	-0.03	-0.06	-0.05	0.06	-0.04	0.07	0.02	-0.07	0.06	0.00	-0.11	0.01	0.09	0.05	**1.00**																																					
17 *Intangible Assets*	0.09	-0.15	0.13	0.17	-0.08	-0.04	-0.03	0.07	0.04	-0.17	-0.04	-0.04	0.07	0.07	0.05	0.04	**1.00**																																				
18 *Tobin's Q*	-0.01	-0.01	0.07	0.05	-0.07	-0.06	0.00	0.08	0.00	-0.09	0.01	-0.21	0.06	0.04	0.00	0.02	-0.01	**1.00**																																			
19 *Risk management Practices*	-0.31	-0.18	0.05	0.13	0.01	-0.24	0.16	0.33	-0.09	-0.04	0.03	-0.01	0.16	0.18	0.03	0.03	0.10	0.04	**1.00**																																		
20 *Assets*	-0.02	0.04	0.02	0.05	0.00	-0.02	0.05	0.06	0.00	0.01	-0.08	-0.05	0.04	0.06	0.01	0.08	0.01	0.01	0.05	**1.00**																																		
21 *EBITDA*	-0.01	-0.02	-0.06	0.01	0.01	-0.01	0.01	0.07	-0.01	-0.01	-0.10	0.10	0.01	0.07	0.02	0.04	0.08	0.02	-0.01	-0.01	**1.00**																																	
22 *R - Square*	-0,02	0,00	-0,16	-0,17	0,12	-0,06	0,11	0,01	-0,12	0,17	-0,03	0,10	0,06	0,04	0,01	0,00	0,02	0,00	-0,01	0,01	0,01	**1.00**																																	
23 *ROE*	0,05	0,12	0,00	-0,03	-0,02	0,06	-0,04	-0,03	0,05	0,04	-0,18	0,04	-0,06	-0,05	0,01	0,00	0,02	-0,01	-0,03	0,00	0,03	0,31	**1.00**																																	
24 *ROC*	0,00	0,04	0,06	0,04	-0,06	0,02	-0,04	0,02	0,06	-0,05	-0,03	-0,03	-0,03	-0,01	-0,07	-0,01	-0,06	0,05	0,00	0,00	0,00	-0,09	0,31	**1.00**																																
25 *Retention Ratio*	-0,01	-0,03	-0,02	-0,01	0,03	-0,02	0,03	0,01	-0,03	0,01	0,10	-0,01	0,02	0,02	-0,23	0,01	-0,07	0,00	0,01	0,00	0,00	-0,08	-0,07	-0,04	**1.00**																															
26 *Depreciation*	-0,01	-0,03	-0,02	-0,01	0,02	-0,02	0,02	0,01	-0,03	0,01	-0,01	0,00	0,02	0,01	0,02	0,01	0,84	0,00	0,01	0,00	0,14	0,08	0,11	-0,05	-0,07	**1.00**																													
27 *Earnings Yield*	-0,01	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,02	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	0,00	0,06	0,06	0,31	-0,01	-0,18	**1.00**																												
28 *Debt / Equity*	-0,06	-0,11	0,04	0,06	-0,02	-0,04	0,00	0,03	-0,02	-0,08	0,02	-0,05	0,03	0,02	0,04	0,02	0,01	-0,08	0,03	0,01	-0,01	-0,01	-0,03	-0,06	-0,11	0,02	-0,01	**1.00**																											
29 *Enterprise Value*	-0,01	0,02	0,01	0,01	-0,01	0,00	0,00	0,01	0,02	0,01	-0,04	0,01	0,02	0,02	0,01	0,01	0,06	0,01	0,01	-0,02	0,01	0,03	0,06	0,01	-0,05	0,04	0,04	0,12	**1.00**																											
30 *Insider Stock Percentage*	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,05	0,01	0,03	0,04	0,00	0,00	0,02	-0,03	**1.00**																										
31 *Board Composition*	-0,36	-0,37	0,04	0,12	0,03	-0,30	0,22	0,33	-0,18	-0,04	0,05	-0,16	0,34	0,22	-0,01	0,04	0,00	-0,03	0,28	-0,01	0,00	-0,02	-0,06	0,08	0,09	0,09	0,06	0,06	0,01	-0,04	**1.00**																									
32 *Compensation / Assets*	-0,01	-0,01	-0,01	0,01	0,02	0,00	0,01	0,01	-0,01	0,00	0,00	-0,01	0,00	0,01	0,00	0,00	0,00	0,00	0,01	0,01	0,00	0,01	0,08	0,03	0,01	0,31	0,02	0,05	0,00	0,10	0,01	**1.00**																						
33 *No. Insiders Holding Stock*	-0,03	-0,14	0,45	0,53	-0,33	0,11	-0,31	0,02	0,28	-0,55	-0,01	-0,35	-0,19	-0,13	0,01	-0,04	-0,02	-0,01	0,02	0,09	0,01	-0,11	-0,05	0,02	-0,04	0,01	-0,07	0,06	0,01	-0,01	0,08	0,12	**1.00**																					
34 *Inventory / Sales*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	-0,02	0,01	-0,01	-0,01	-0,01	0,00	-0,01	0,00	-0,05	-0,07	-0,01	0,01	-0,03	0,03	0,03	-0,02	0,00	0,03	-0,04	-0,05	**1.00**																				
35 *Net Oper. Margin*	0,00	-0,01	-0,01	-0,01	0,01	0,00	0,00	0,00	-0,01	0,00	-0,01	0,00	-0,01	-0,01	0,00	0,00	0,01	-0,02	-0,01	0,01	0,00	0,09	0,19	0,31	0,02	0,05	0,08	0,22	0,07	0,01	0,00	0,21	0,01	-0,11	**1.00**																			
36 *Dividend Growth*	-0,01	0,02	0,01	-0,01	-0,02	0,00	-0,01	0,01	0,02	0,00	-0,08	0,02	0,03	0,03	0,06	-0,01	0,04	-0,01	0,01	0,00	0,00	0,06	0,12	0,09	-0,14	0,02	0,04	0,00	0,06	0,01	0,06	-0,01	-0,03	-0,04	0,07	**1.00**																		
37 *FCFF*	0,01	0,01	0,00	0,00	0,00	0,01	0,00	-0,01	0,01	0,00	-0,01	0,00	-0,01	0,00	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,23	0,11	0,01	-0,12	0,03	0,29	0,07	0,22	0,00	0,05	0,01	-0,06	0,01	0,00	0,03	**1.00**																	
38 *FCFE*	0,01	0,01	0,00	0,00	0,00	0,01	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,12	0,04	0,21	-0,01	0,02	0,22	0,34	-0,08	0,00	0,00	0,00	-0,07	0,00	0,01	0,01	0,72	**1.00**																
39 *Cap. Expenditure*	0,01	0,01	0,00	0,00	0,00	0,01	0,00	-0,01	0,01	0,00	0,00	0,00	-0,01	0,00	0,03	0,00	0,00	0,00	-0,01	0,00	0,00	0,23	0,05	0,17	-0,01	0,31	-0,03	0,05	-0,06	0,00	-0,06	0,01	-0,06	0,00	0,00	0,01	0,68	0,61	**1.00**														
40 *WACC*	-0,06	-0,02	0,01	0,01	-0,01	-0,04	0,02	0,06	0,00	0,00	0,03	-0,01	0,05	0,04	-0,03	-0,01	0,00	0,03	0,03	0,01	0,01	0,05	-0,12	0,01	0,01	0,11	-0,11	-0,08	0,07	0,02	0,03	0,08	0,01	-0,10	0,06	0,00	0,01	0,03	0,01	**1.00**										
41 *Cost of Equity*	-0,06	-0,07	-0,03	-0,01	0,03	-0,07	0,06	0,06	-0,05	0,02	0,05	-0,01	0,07	0,05	-0,01	0,00	0,01	0,01	0,03	0,01	0,00	0,04	-0,01	0,17	0,02	0,19	-0,01	0,01	0,14	0,01	0,05	0,16	0,07	-0,01	0,03	0,03	0,02	0,02	0,02	0,68	**1.00**								
42 *Cost of Debt*	-0,08	0,05	0,14	0,09	-0,15	-0,01	-0,07	0,08	0,12	-0,06	-0,04	-0,02	0,07	0,04	-0,01	0,03	0,03	-0,05	0,06	0,02	0,03	-0,03	0,00	-0,01	0,00	0,01	0,00	0,30	0,05	0,00	0,05	-0,01	0,03	0,01	0,00	0,01	0,01	0,01	0,01	0,02	-0,34	**1.00**							
43 *Z - Score*	0,01	0,01	0,00	0,00	0,01	0,01	-0,01	-0,02	0,00	0,00	0,03	0,00	-0,02	-0,03	-0,04	-0,01	-0,01	0,02	-0,01	0,00	0,00	0,00	0,00	0,01	0,05	-0,01	0,00	-0,02	-0,02	0,00	-0,01	0,00	0,01	0,00	0,00	-0,01	0,00	0,00	0,00	0,00	0,09	-0,34	**1.00**						
44 *MPK*	0,00	-0,01	-0,01	-0,01	0,00	0,00	0,00	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,09	-0,01	**1.00**					
45 *ROA*	-0,01	-0,01	0,00	0,00	0,00	-0,01	0,01	0,01	0,00	0,00	0,00	0,00	0,02	0,03	0,00	0,00	0,00	-0,02	0,01	0,01	0,00	0,01	0,01	0,01	0,00	0,00	0,00	0,01	0,00	0,00	0,01	0,00	0,01	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,04	0,00	0,03	-0,02	**1.00**				
46 *IK*	-0,01	-0,01	0,00	0,00	0,00	-0,01	0,01	0,00	-0,01	0,00	-0,01	0,00	0,00	0,00	0,02	0,01	0,00	-0,02	0,00	0,00	0,00	0,01	-0,01	-0,01	0,00	0,00	0,00	0,11	0,03	0,00	0,01	0,00	-0,01	0,01	0,00	0,00	0,00	0,00	0,00	-0,01	-0,01	0,04	-0,03	0,05	0,01	**1.00**			
47 *Alpha*	0,00	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	0,00	0,02	0,00	-0,01	**1.00**		
48 *Beta*	0,00	0,00	-0,01	0,00	0,01	0,00	0,00	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,00	-0,01	0,00	0,00	0,00	-0,02	0,00	0,01	-0,01	0,02	-0,05	-0,03	**1.00**	
49 *Total Beta*	0,02	0,03	0,04	0,04	-0,04	0,03	-0,05	-0,02	0,04	-0,04	0,02	-0,01	-0,03	-0,03	-0,01	0,00	-0,01	0,01	-0,01	0,00	0,00	-0,15	-0,01	0,00	0,01	0,00	0,00	-0,01	-0,02	0,00	0,00	0,00	0,02	0,00	-0,01	-0,01	0,00	0,00	0,00	-0,02	-0,02	-0,02	-0,04	-0,03	-0,04	-0,03	-0,04	0,10	**1.00**

Table 12: UK (AIM) Correlation Matrix

Variables	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
1 *Consumer Confidence Index*	**1.00**																																																
2 *GDP Growth*	0.62	**1.00**																																																
3 *Short - Term Interest Rate*	0.20	0.10	**1.00**																																														
4 *Long - Term Interest Rate*	0.30	0.13	0.69	**1.00**																																													
5 *Yield Spread*	-0.05	-0.05	-0.65	-0.30	**1.00**																																												
6 *ICRG Political Risk Rating*	0.41	0.34	-0.33	-0.07	0.44	**1.00**																																											
7 *Unemployment Rate*	-0.62	-0.42	-0.71	-0.52	0.65	-0.01	**1.00**																																										
8 *Inflation*	-0.02	0.32	0.61	0.50	-0.52	-0.18	-0.42	**1.00**																																									
9 *Output Gap*	0.55	0.45	0.76	0.67	0.57	-0.03	-0.79	0.61	**1.00**																																								
10 *Financial Crisis*	-0.58	-0.52	-0.48	-0.67	0.19	-0.41	0.53	-0.44	-0.63	**1.00**																																							
11 *Volatility*	-0.25	-0.28	-0.02	-0.05	0.01	-0.01	0.10	-0.04	-0.08	0.09	**1.00**																																							
12 *JOBS Act*	0.19	0.16	-0.55	-0.64	0.24	0.03	0.05	-0.49	-0.42	0.41	-0.13	**1.00**																																							
13 *FAS 123R*	-0.51	-0.48	-0.16	-0.43	-0.10	-0.67	0.34	-0.31	-0.51	0.58	0.05	0.32	**1.00**																																							
14 *Entry Amendments*	-0.30	-0.23	-0.76	-0.72	0.63	0.05	0.66	0.62	-0.72	0.67	-0.01	0.61	0.52	**1.00**																																							
15 *Effective Tax Rate*	0.02	0.01	-0.03	-0.05	0.31	0.03	-0.02	-0.04	-0.01	0.02	0.01	0.08	0.01	0.03	**1.00**																																							
16 *Big 4 Auditors*	0.23	0.02	0.19	0.23	0.03	0.42	-0.30	-0.05	0.35	-0.52	0.10	-0.31	-0.40	-0.35	0.01	**1.00**																																							
17 *Intangible Assets*	0.02	-0.03	-0.08	-0.09	0.05	-0.01	0.04	-0.07	-0.08	0.09	-0.12	0.08	0.07	0.09	0.03	-0.01	**1.00**																																						
18 *Tobin's Q*	0.04	0.03	0.00	0.02	0.01	0.05	-0.02	-0.04	0.01	-0.01	0.03	0.03	-0.07	0.00	-0.01	-0.02	-0.20	**1.00**																																						
19 *Risk management Practices*	0.06	-0.03	0.06	0.09	-0.02	0.21	-0.01	0.04	0.07	-0.13	0.04	-0.16	-0.07	-0.09	0.01	0.28	-0.01	-0.01	**1.00**																																					
20 *Assets*	0.18	0.07	-0.06	-0.07	0.03	0.01	0.03	-0.04	-0.05	0.05	-0.30	0.07	0.04	0.07	0.00	-0.03	0.44	-0.14	-0.01	**1.00**																																					
21 *EBITDA*	-0.02	0.00	0.07	0.01	0.05	0.07	-0.02	0.06	0.02	0.06	0.02	-0.01	-0.02	-0.02	-0.04	0.00	0.03	0.08	0.01	0.16	**1.00**																																				
22 *R - Square*	-0,07	-0,04	-0,11	-0,07	0,11	0,05	0,15	-0,05	-0,11	0,01	-0,01	-0,01	-0,02	0,09	0,00	0,01	0,07	-0,01	0,01	0,07	0,01	**1.00**																																				
23 *ROE*	0,02	0,03	0,01	0,01	-0,01	0,01	-0,01	0,02	0,02	-0,02	-0,18	-0,01	-0,02	-0,01	0,00	-0,01	0,06	-0,08	0,00	0,17	0,01	0,18	**1.00**																																				
24 *ROC*	0,01	0,01	0,04	0,04	-0,03	-0,01	-0,02	0,04	0,03	-0,03	-0,05	-0,03	-0,02	-0,04	0,00	0,00	0,00	-0,04	0,01	0,05	0,01	0,02	0,22	**1.00**																																				
25 *Retention Ratio*	-0,01	0,00	0,00	0,02	0,01	0,01	0,02	0,01	0,00	-0,01	0,11	-0,04	-0,01	-0,01	0,00	0,00	-0,01	0,04	0,01	-0,16	-0,01	0,20	-0,01	-0,02	**1.00**																																				
26 *Depreciation*	0,00	0,00	0,00	0,01	0,00	0,00	-0,01	0,01	0,01	-0,01	-0,02	0,00	-0,01	-0,01	0,00	0,01	0,07	0,00	0,00	0,13	0,91	0,11	0,07	0,02	-0,01	**1.00**																																				
27 *Earnings Yield*	0,03	0,02	0,01	0,02	0,00	0,01	-0,01	0,00	0,01	-0,02	-0,17	0,00	-0,02	-0,01	0,00	-0,01	0,02	0,00	0,00	0,02	0,00	0,04	0,05	0,01	-0,05	0,00	**1.00**																																				
28 *Debt / Equity*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,00	-0,01	-0,09	-0,01	0,03	-0,01	**1.00**																																				
29 *Enterprise Value*	0,02	0,01	-0,04	-0,05	0,02	0,00	0,00	-0,03	-0,03	0,03	-0,11	0,06	0,03	0,04	0,00	-0,02	0,23	0,02	-0,01	0,33	0,23	0,02	0,04	0,01	-0,03	0,13	0,09	0,03	**1.00**																																				
30 *Insider Stock Percentage*	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	0,00	0,00	0,01	0,01	0,02	-0,01	-0,29	-0,01	-0,04	**1.00**																																				
31 *Board Composition*	-0,01	0,00	0,00	-0,01	-0,01	0,00	0,00	-0,02	-0,01	0,01	-0,01	0,01	0,01	0,01	0,00	0,01	0,00	0,00	0,06	0,00	0,00	0,05	0,01	0,04	0,04	0,01	0,04	0,04	0,00	0,04	0,03	**1.00**																																				
32 *Compensation / Assets*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	0,00	0,00	-0,09	-0,04	0,02	-0,02	-0,02	0,00	0,03	0,00	0,05	**1.00**																																				
33 *No. Insiders Holding Stock*	-0,01	0,00	0,00	-0,01	-0,01	0,01	0,01	0,01	0,00	0,00	0,00	-0,02	0,00	0,00	0,00	0,00	-0,07	-0,04	0,02	0,04	0,00	-0,19	-0,09	-0,09	-0,07	-0,03	0,04	0,00	-0,02	0,01	0,02	0,00	**1.00**																																				
34 *Inventory / Sales*	0,00	0,00	0,00	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,00	-0,02	0,00	0,05	0,18	-0,09	-0,01	0,00	-0,08	-0,05	-0,03	0,01	0,11	-0,01	0,03	**1.00**																																				
35 *Net Oper. Margin*	-0,01	0,00	0,00	0,00	0,01	0,00	0,01	0,01	0,00	-0,01	-0,02	-0,01	0,00	0,00	0,00	0,00	0,01	-0,02	0,00	0,03	0,00	0,02	0,03	0,01	0,00	0,01	0,06	0,00	0,04	0,00	-0,03	0,00	0,04	-0,25	**1.00**																																				
36 *Dividend Growth*	0,04	0,04	0,01	0,00	-0,02	0,00	-0,04	0,03	0,03	-0,03	-0,03	0,03	-0,02	-0,02	0,00	-0,02	0,01	0,01	0,00	0,00	0,01	0,13	0,14	0,04	0,06	0,03	0,09	-0,01	0,02	0,00	0,01	0,00	0,21	-0,05	-0,01	**1.00**																																				
37 *FCFF*	-0,01	-0,01	-0,01	-0,01	0,01	0,00	0,01	-0,01	-0,01	0,01	-0,01	0,00	0,00	0,01	0,00	0,00	0,04	0,00	0,00	0,08	0,34	0,08	0,13	0,01	-0,07	0,27	-0,07	0,00	0,15	0,00	0,05	-0,02	-0,07	-0,03	0,00	0,07	**1.00**																																				
38 *FCFE*	0,00	0,00	-0,01	-0,01	0,01	0,00	0,01	-0,01	-0,01	0,01	-0,02	0,01	0,01	0,01	0,00	0,00	0,04	0,02	0,00	0,07	0,30	0,06	0,09	0,03	-0,02	0,28	0,01	0,02	0,14	0,00	0,04	-0,01	-0,05	-0,06	0,09	0,07	0,39	**1.00**																																				
39 *Cap. Expenditure*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	0,00	0,00	0,02	0,00	0,00	0,01	0,00	-0,01	-0,07	0,00	0,00	-0,14	-0,90	-0,01	-0,12	-0,02	0,08	0,63	-1,19	-0,03	-0,15	0,01	-0,04	0,02	0,03	-0,08	0,01	-0,08	-0,36	-0,27	**1.00**																																				
40 *WACC*	-0,04	-0,02	0,04	0,05	-0,02	-0,02	0,03	0,04	0,01	-0,02	0,02	-0,07	-0,01	-0,03	0,00	0,00	0,02	0,04	0,00	-0,05	-0,01	0,05	-0,02	-0,02	0,07	-0,05	0,51	-0,08	-0,06	-0,07	0,03	0,00	-0,04	0,08	-0,03	0,17	0,00	0,00	0,04	**1.00**																																				
41 *Cost of Equity*	-0,11	-0,08	0,01	0,01	0,00	-0,03	0,09	0,01	-0,04	0,02	0,06	-0,07	0,02	0,01	0,00	-0,01	0,05	0,01	0,00	0,05	0,00	-0,17	0,16	0,07	-0,04	0,05	0,01	-0,07	0,03	-0,03	-0,09	0,00	-0,02	-0,28	-0,02	0,06	-0,04	0,03	-0,03	0,88	**1.00**																																				
42 *Cost of Debt*	0,15	0,07	0,38	0,42	-0,23	-0,05	-0,27	0,22	0,36	-0,28	0,00	-0,29	-0,16	-0,36	-0,01	0,15	0,02	-0,05	0,04	0,10	0,00	-0,04	-0,03	0,01	-0,04	0,00	-0,01	0,02	0,00	0,00	0,00	0,00	0,02	0,00	0,00	-0,02	0,00	-0,01	0,00	-0,03	0,01	**1.00**																																				
43 *Z - Score*	0,00	0,00	0,00	0,00	0,01	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,95	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	**1.00**																																				
44 *MPK*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	**1.00**																																				
45 *ROA*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,02	-0,73	0,00	**1.00**																																				
46 *IK*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	0,00	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,02	0,06	0,05	0,01	**1.00**																																				
47 *Alpha*	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00	-0,07	-0,01	0,00	0,00	-0,02	**1.00**																																				
48 *Beta*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,00	0,00	0,00	-0,05	0,00	-0,01	0,00	0,00	-0,01	**1.00**																																				
49 *Total Beta*	0,02	0,00	0,02	0,01	-0,03	-0,01	-0,04	0,00	0,02	0,00	0,01	0,00	0,01	-0,02	0,00	0,00	-0,01	0,00	0,00	-0,01	0,00	-0,17	-0,01	-0,02	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	-0,02	-0,17	-0,05	0,05	-0,05	0,05	0,00	0,13	**1.00**

Table 13: US (NASDAQ) Correlation Matrix

The correlation matrix does not reveal any irregularities. We will, however, be revisiting the correlation as a topic when we discuss the proposed methodology for the thesis.

3.4.3. Size as a matching factor

As has already been explained in several circumstances in the thesis so far, the primary focus of this study are private enterprises. Earlier research, has avoided focusing on them until recently as there was a lack of available data, which diverted researchers towards finding proxies for the private companies they wanted to draw inference on (St.-Pierre and Bahri, 2006). The common characteristic however, in the papers on private businesses, is their approach on how to match the private companies with their public counterparts, namely the size of the firms (see for example the paper of Rijken et al., 1999).

The size can be defined through different means. It can be approximated through revenues (Jensen and Murphy, 2004), assets (Lin and Dongwei, 2008), or the number of employees (Höglund and Sundvik, 2016). We follow the ideas set forth in several studies, with the most prominent ones being those of (Cooper and Priestley, 2016; Gerakos et al., 2013; Livingston, 2014) and match the AIM, NASDAQ and private companies by the assets or if that was not possible (in rare cases as databases report that element more often than not) by the number of employees. In this way, we make sure that the results we would transfer from the public enterprises over to the private ones are not over or understated due to the companies' size.

3.4.4 Expected impact of individual variables, on our components

Having conducted the analysis of the data sets and the variables that we will be using to determine how the discount rates are affected by linear combinations of the variables described in the previous section. We will tabulate the original variables, explaining the predicted significance of each variable on the components; however, we must keep in mind how PCA alters the original data. We will discuss it extensively in the section that follows, however, PCA is a size reduction technique, namely it reduces the size of large samples of data to smaller more comprehensive ones, with a decreased number of variables,

while simultaneously retaining the largest part of the variation possible. These new determinants are weighted linear combination of the underlying variables and in order to understand how they impact the valuation (through the multiple of P/E ratio we will be using as its proxy), we need to conduct an analysis of how the original variables contribute to the creation of the new ones.

Following, however the methodology of similar papers (Boone, Casares Field, Karpoff, and Raheja, 2007; Callahan, Millar, and Schulman, 2003; Madrid-Guijarro, Garcia, and Auken, 2009), in the ensuing table (Table 14) we postulate the significance of the primary variables:

	Valuation Multiple	**Impact on the Valuation Multiple (P/E Ratio)**
Variable	UK / US	UK / US
Consumer	Significant / Significant	A higher valuation is associated with a higher CCI score
GDP Growth	Significant / Significant	A higher GDP Growth will be positively associated with a higher valuation multiple
Short - Term Interest Rates	Significant / Significant	Lower short-term interest rates will be positively contributing to the valuation measure
Long - Term Interest Rates	Not Significant / Not Significant	Lower long-term interest rates will be positively associated with the valuation multiple
ICRGC Rating	Significant / Significant	A higher ICRGC rating will be positively associated with a higher valuation multiple
Unemployment Rate	Not Significant / Not Significant	A lower unemployment rate will affect the valuation positively
Inflation	Not Significant / Not Significant	A lower inflation rate will be positively associated with the company's valuation
Output Gap	Not Significant / Not Significant	Lower Output Gap is positively associated with a higher valuation
Financial Crisis	Significant / Significant	The Financial Crisis affected the valuation multiple negatively
Volatility	Not Significant / Not Significant	Higher volatility will be negatively affecting the valuation multiples
JOBS Act	- / Significant	This legislative act will affect positively the valuation of the company's valuation
FAS 123R	- / Significant	This legislative act will affect positively the valuation of the company's valuation
Entry Amendments	- / Significant	This legislative act will affect negatively the company's valuation
IAS 34	Significant / -	This legislative act will affect positively and increase the valuation of the company's valuation
Companies Act 2006	Significant / -	This legislative act will affect positively and increase the valuation of the company's valuation

IAS Framework Adoption	Significant / -	This legislative act will affect positively and increase the valuation of the company's valuation
Effective Tax Rate	Significant / Significant	Higher Effective Tax Rate is linked to lower valuations
Big 4 Auditors	Significant / Significant	Big 4 Auditors auditing the company leads to a higher valuation
Intangible Assets	Significant / Not	Higher intangible asset value is linked to a higher valuation
Tobin's Q	Not Significant /	A higher Tobin's Q is linked to a higher valuation
Risk management Practices	Not Significant / Significant	The existence of management practices is associated with a higher valuation
Assets	Significant / Significant	Higher assets will be positively associated with a higher valuation
EBITDA	Not Significant /	Higher EBITDA is associated with a higher valuation
R - Square	Not Significant / Not	Higher R-Square is associated with a higher valuation
ROE	Not Significant /	Higher ROE is associated with a higher valuation
ROC	Significant / Not	Higher ROC is associated with a higher valuation
Retention Ratio	Not Significant / Not	Higher retention ratio is associated with a higher valuation
Depreciation	Not Significant / Not	Lower depreciation is associated with a higher valuation
Earnings Yield		A higher EY is associated with a higher valuation
Debt / Equity	Significant / Significant	A higher Debt / Equity ratio will be positively associated with a higher valuation
Enterprise Value	Not Significant / Not Significant	A higher EV will be positively associated with a higher valuation
Insider Stock Percentage	Significant / Significant	A higher number of directors holding company shares will be positively associated with a higher valuation
Board Composition	Not Significant / Not Significant	A better board composition (balanced number between inside and outside directors) will be positively associated with the valuation multiple
Compensation / Assets	Significant / Significant	Higher Compensation / Assets ratio is positively associated with a higher valuation multiple
No. Insiders Holding Stock	Significant / Significant	Lower no. of insiders holding stock are positively associated with a higher valuation
Inventory / Sales	Significant / Significant	Higher inventory / sales ratio will affect positively the valuation multiple
Net Oper. Margin	Significant / Significant	Higher net operating margin will contribute positively to the valuation multiple
Dividend Growth	Not Significant / Not Significant	A higher dividend growth potential will be positively associated with the valuation multiple
FCFF	Significant / Not Significant	Higher free cash flow to the firm are positively associated with the valuation multiple
FCFE	Not Significant / Significant	Higher free cash flow to equity are expected to negatively affect the valuation multiple
Cap. Expenditure	Significant / Significant	Higher Cap. Expen. is positively associated with a higher valuation multiple
WACC	Significant / Significant	A lower WACC will be positively associated with the valuation multiple

Cost of Equity	Not Significant / Significant	A lower cost of equity will be positively associated with the valuation
Cost of Debt	Significant / Not Significant	A lower cost of debt will be positively associated with the valuation
Z - Score	Significant / Significant	A higher Z-Score will be positively associated with the valuation multiple
MPK	Not Significant /	A higher MPK ratio will increase the valuation measure
ROA	Significant / Significant	A higher ROA will increase the valuation measure
IK	Not Significant /	A higher IK ratio will increase the valuation measure
Alpha	Significant / Significant	A higher alpha is expected to positively affect the valuation measure
Beta	Not Significant / Significant	A higher Beta will be negatively associated with the valuation multiple
Total Beta	Significant / Not Significant	A higher total Beta will be negatively associated with the valuation multiple

Table 14: This table reflects our expectations on the effect of the original variables on the (P/E ratio) measure of valuation, as it will be approximated by the new variables constructed through the PCA. The significance we predict in this table is based not only upon the relevant literature, but it also represents the considered view on how our original variables will contribute to the components that will be used in the panel regression analysis we will be performing after the PCA. The idea behind assigning the significance (as this is displayed in the first column of the table) is how the UK and US companies in the sample represent the private companies in terms of the liquidity constraints they face, management and agency issues that might arise, as well as a lack of available information on them.

Dependent Variable

As we have referred several times already to the P/E ratio that will be used as the valuation proxy in this thesis, it is imperative that we review its properties, as well as explain the rationale for its selection. The P/E ratio is one of the valuation multiples most commonly employed, especially for relative valuation (Kim and Ritter, 1999). Earnings is one of the most closely followed accounting information elements especially for private companies. Lee and Masulis (2011) argue that during the process of private companies going public, and in order to set the price for the IPO, underwriters set the offering price by relying on a combination of industry price-earnings ratios and the private companies own earnings. The importance of earnings, and subsequently the related ratios that are used in the valuation process, can also be underlined by the fact that it has spurred a separate strand of the literature to grow regarding this issue (see for example the survey paper of Dechow et al., 2010, who summarize the earnings quality literature and explain the increased investor responsiveness to earnings-related news). Other measures have been considered (for

instance the P/B ratio), however the consensus in the literature is that the P/E ratio can express the valuation process more accurately than the other ratios (Cheng and McNamara, 2000).

The generalized form of the P/E ratio can be defined as:

$$\frac{P}{E} = \frac{Market\ Price\ per\ Share}{Earnings\ per\ Share} \tag{4.1}$$

Damodaran (2002), indicates that in this calculation, the market price per share is consistently used in the estimations of the related ratios, however the variations of the earnings estimations (current, trailing, forward, diluted and primary), pose a problem, especially for firms that are characterized by increased growth (which is something we expect for the AIM firms, as the majority of them are young, growth firms). Current, trailing and forward are the most prevalent earnings measures that are used, as can be seen in several studies (Alford, 1992; Anderson and Brooks, 2006; Kim and Ritter, 1999). For the purposes of this thesis we will be following Elnathan et al. (2010), and use the most current estimates from each company's financial statements.

A major point that should be noted is that the P/E ratio can be expressed (in the case of forward earnings) as a function of the payout ratio, the company's expected growth and its cost of equity. This is particularly important, as it ties the P/E ratio to the return-on-equity, as the payout ratio equals the fraction of 1 minus the growth rate over the company's return-on-equity. As growth firms often reinvest their earnings to further spur growth (Dargenidou, McLeay, and Raonic, 2007), they either reduce the amount they pay or do not pay dividends at all. The connection, however, between payout ratio and earnings allows us to replace the free cash flow to equity (FCFE) with the payout ratio. To provide a better understanding on how all

these measures are linked together, we appraise the construction of the price to earnings ratio (P/E) through the following formulas[34]:

$$Value\ of\ Equity\ (EV) = \ P_0 = \frac{DPS_1}{k_e - g_n} \qquad (4.2)$$

In which case k_e is the cost of equity and g_n is the expected growth rate of the company. We divide equation (4.2) with the earnings (E) and the resulting equation is the discounted cash flow equation for a stable growth firm:

$$\frac{P}{E} = \frac{P_0}{EPS_0} = \frac{payout\ ratio * (1 + g_n)}{k_e - g_n} \qquad (4.3)$$

Growth and P/E ratios are intertwined at a high degree. Damodaran (2005), remarks that the P/E ratio is responsive to growth variance especially in times when interest rates are high rather than low. This is attributed to the fact that the PV of future cash flows is affected by the investment's growth rate and the interest rates of those investments. However, as interest rates increase, growth is suppressed, which ultimately leads to lower present values for the investments. Besides growth, the P/E ratio has also been established to be linked with another significant part, of this thesis' focus, the discount rates (see for example the studies of Berkman, Bradbury, and Ferguson, 2000; Cochrane, 2011; Goddard, McMillan, and Wilson, 2009). The main notion underlying the various studies on the topic is that there is an inverse relationship between the risk of a firm (as it is reflected by its Beta) and the P/E ratio.

A major concern throughout the literature is the presence of negative earnings within the samples. There are several reasons as to why negative earnings exist and they are all linked to the underperformance of companies. Damodaran (2012) categorizes them in short-term, or temporary and long-term problems. In the former category we have those reasons that are primarily associated with the operations of the firm,

[34] A thorough analysis of the aforementioned formulas can also be found in: http://pages.stern.nyu.edu/~adamodar/New_Home_Page/invfables/peratio.htm

namely product recalls, strikes, product cyclicality and industry-related issues (for instance hospitality and leisure-time related companies in areas that are affected by extraordinary circumstances such as extreme weather conditions). In the latter category, negative earnings are related to factors that focus on management issues, for example due to poor strategic and marketing decisions.

The focus on negative earnings is understandable if one delves further into the problems that arise because of them and the valuation process. As we have already explained the earnings are closely related to both the company's growth estimations, as well as to the cost of equity of the company. Negative or zero earnings do not allow appraisers to estimate these two inputs which are fundamental to the value calculation. In addition to that, valuations are performed with the underlying assumption that a company has an infinite life in mind. This assumption, however, cannot be made for companies that exhibit negative earnings. Finally, as Damodaran (2012) explains, negative earnings impede the appraisers' ability to compute the tax-related measures, which can be used in valuations, such as the after-tax operating income.

To address the issue of negative P/E, resulting from negative earnings, we follow the methodology proposed by Elnathan et al. (2010), who suggest a log-linear specification modelling for negative earnings (as those are represented by the P/E ratio for the purpose of this thesis), which can be defined as:

$$f(x) = \begin{cases} LN(x+1), & x \geq 0 \\ -LN(-x+1), & x < 0 \end{cases} \tag{4.5}$$

The above specification suggests that the variable under examination is log transformed and as the function is monotone allows for the preservation of all the information included in the original variable. To further elaborate on that notion, the P/E ratio can be defined in both cases that earnings are either zero or negative, without requiring us to dismiss the negative values, and thusly losing datapoints (as the number within the function becomes positive even when the P/E is negative the second branch of the function turns it into a positive number), while maintaining the direction of the curve that the sign dictates). Similar methodologies are being applied in other papers, such as those of Draper and Paudyal (2008) and Ritter (2014).

Having defined the original hypotheses and the variables we will be using in this thesis, we can now proceed to explain how the proposed methodology will work and how we will be linking all the variables together to attempt and create an integrated framework that can be used for valuing businesses. As we explained, previously, however these variables will be used to create a new index of variables, through the novel methodology applied in this study, that will help us achieve this goal, and thusly these hypotheses will be tested under a different prism.

3.5 Tools and Techniques Employed

3.5.1 Previously used methodologies as explained in the relevant literature

Our analysis is spurred by studies such as the survey paper of Cochrane (2011), who suggests among others that the main focus of research has shifted from asset-pricing models to discount rate research. He argues that discount rates should be examined under a new prism namely one that accounts for both shifts over time, as well as, changes in asset class influences. The methodologies that are most commonly employed by practitioners, do not allow for that as will become apparent in the sections that follow. Specifically, linear regression analysis (best expressed through the CAPM and its variations) has been heavily criticized for a number of reasons, that have to do mainly with how it allows for large portions of the expected returns' cross-section to go unexplained, as well as for its inability to explain over-time variance of the discount rates (Booth, 1999).

Factor analysis has also been part of the literature for several years. Arbitrage Pricing Theory (Ross, 1982) is a prime example of this methodology. Also, the weaknesses of CAPM, and linear regression analysis in general, led Fama and French (1992), to develop their three factor model that accounted for both size and market value. This approach is used to represent a large amount of data with a smaller number of variables called factors. Its main attributes are that factors, and the error terms are zero-mean variables, and that error terms and factors are uncorrelated. However, the main disadvantages, are that the factors are unobserved

variables, which means that it is impossible to accurately define them, the number of factors cannot be accurately determined, a problem that is of utmost importance in approximate factor models, and the methodology itself is based on the premise that the data are determined by a distribution and an underlying model, which is not always a realistic assumption.

In this study the methodology that will be used is the Principal Components Analysis (PCA), which is closely related to factor analysis but with some major differences that will be considered in the sections to follow. This methodology is used to reduce the size of large datasets, by retrieving those linear combinations of the variables that make the most significant contribution to the total variability of the sample. The components derived from the process, are ranked in a descending order based on the percentage of the variability they are responsible for. Through applying several criteria, those components with the lowest contribution can be excluded. This technique shares similarities with factor analysis, but unlike it, PCA does not assume any underlying data structure, which allows for more complex datasets to be used. As will be exhibited, the proposed methodology has several critical advantages over its predecessors, while avoiding the weaknesses that make the previous techniques employed less appropriate for determining discount rates.

The rest of this section is structured as follows. First, we explain linear regression analysis, not only because it is the most commonly used methodology but also because it will also be employed at a later stage in the research (for reasons that will be explained in a following chapter). We will consider in depth how it works, the most renowned case application (CAPM) in the finance literature and the criticism of it. Then there will be an analysis of factor analysis, as this was the next area of focus in relevant studies, by explaining and contrasting the differences with multiple regressions, how the method is conducted and what its disadvantages are. The conclusion will describe the main methodology which will be applied in this study, explaining how it was developed and point out the reasons as to why it is chosen, followed by a detailed description of the model applied. Finally, we will explain how we will test the validity of the results, not only on the original dataset but also on the unique dataset of private enterprises.

3.5.2 Linear Regression Analysis

Linear Regression Analysis (LRA), is probably the oldest and most commonly used methodology for studies of a cross-sectional nature, not only in finance but also most other scientific fields. The rationale of the approach is to determine how an independent variable (or variables) affect the dependent variable as they change. It does that by estimating the mean value of the dependent in terms of a set of known values for the explanatory variables. Another way of thinking of this, is that LRA attempts to find all those points that create a line that best explains how the average point in the dataset responds to changes in the controlled variable. However as Gujarati and Porter (2009) argue, dependence does not suggest causation, it is the responsibility of the researcher and the underlying theories to determine any possible causal relationships.

In its simplest form a regression analysis (the univariate analysis) can be expressed as:

$$Y_i = \hat{a}_1 + \hat{\beta}_1 X_i + e \tag{4.6}$$

Where Y_i is the dependent variable, $\hat{a}_1$ is the expected value of the constant term, $\hat{\beta}_1$ is the coefficient that expresses the impact of X_i on the dependent variable, as well as the direction of the associated independent variable, X_i is the independent variable and e is the error term. It is also important to know that this technique is governed by several assumptions that should not be violated or the estimators may become biased.

In finance and accounting however, we are often required to examine the relationship of more than one explanatory variable with a predicted one[35]. In this case, we use a Multiple Linear Regression (MLR) model, which can be written as follows[36]:

$$Y_i = a + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_n X_n + e \tag{4.7}$$

[35] For example, a family of variables consisting of company characteristics.

[36] In its matrix form is written as $Y = \beta X + e$

Where Y_i is the dependent variable, a is the constant, β are the coefficients, X are the independent variables and finally e is the error term, which can be also written as $Y_i - \hat{Y}_i$, with $\hat{Y}_i$ being the estimation for the explained variable. The main goal of regression analysis is to minimize the error with the additional restraint of the coefficients, a notion that can be expressed as:

$$min \sum_{i=1}^{n} \left(Y_i - \hat{Y}_i\right)^2 \tag{4.8}$$

ere $\hat{Y}_i$ is the expected value of Y_i. As noted, there are some assumptions underlying this methodology. Firstly, the mean of the error term is zero, and its variance is σ^2. Furthermore, each error term is uncorrelated with the other error terms, as well as, with the independent variables and All these assumptions lead them to be i.i.d (independently and identically distributed). Although these assumptions seem logical in a theoretical setting, i.i.d is not plausible in the real world and has been the epicenter of criticism on linear regression analysis (Cochrane, 2011; Fabozzi, Focardi, Rachev, and Arshanapalli, 2014; Gujarati and Porter, 2009).

3.5.3 Goodness of fit and Hypothesis Testing

By design, MLR requires us to determine a set of explanatory variables that is correlated to the dependent variable, while excluding all those that are not. This might prove difficult as, in many cases, out of concern for including non-important variables, researchers omit significant ones. The contrary can also happen, where less important variables are included, which will affect the coefficient of determination R^2. Moreover, the quality of the estimation is largely dependent on the sample size. More extensive datasets provide higher quality results.

R^2 is particularly important as it reveals, how much of the variability of the dependent variable is explained by the model. Low R squares, from MLR models such as the CAPM (which will be considered more extensively in the section that follows), dictated the need to turn to other forms of analysis (Cochrane, 2011). Additionally, high R^2 are not always an indication of a high explanatory power of the model. This

stems from the fact that adding variables artificially inflates R^2 and can lead to ambiguous results. For that reason, the adjusted $\overline{R^2}$ has been introduced, which is a variant of the original, that taxes the model for the inclusion of extra variables, by reducing its value as a response to every new addition. One can also conduct an F-test to determine whether the insertion of another variable is improving the overall fit of the model.

To examine the significance of the model, a series of tests can be utilized. The most important one is the t-test, which is used to determine whether an independent variable's coefficient is significant or not. This test is conducted by first determining the standard error of each coefficient, which can be calculated as an approximation of the variance matrix of all the coefficients. The results are then contrasted with a critical value that is linked to the level of significance[37]. The p-values, originating from the t-test, need to be less than the level of significance set as threshold, for the explanatory variables to be important in the explanation of the dependent variable.

3.5.4 Regression Analysis and the CAPM

The CAPM is the most commonly used methodology by practitioners throughout the latter half of the last century. It is a model used in asset pricing, and the main idea was to determine a factor that would act as a measure for the covariance of an asset with the market portfolio, or to put it more aptly for the asset's undiversifiable, or as it is more commonly referred to, systematic risk. The model itself is expressed as:

$$R_a = R_f + \beta_\alpha(R_m - R_f) \tag{4.9}$$

Where R_a is the asset's returns expressed as a linear function of the risk-free rate R_f and the product of the asset's measure to systematic risk β_α, and the market risk premium $(R_m - R_f)$, which is indicated as the difference of the market returns R_m and the risk-free rate R_f. The expression above is simply a linear

[37] Denoted with *a*.

regression where $R_m - R_f$ is the independent variable. The above expression can be re-written, to include the time variation (denoted as *t*) aspect of it, as:

$$R_{a,t} = R_{f,t} + \beta_{\alpha,t}\left(R_{m,t} - R_{f,t}\right) + e_{i,t} \quad (4.10)$$

Where all the above remains as explained before, but with the inclusion of an error term, besides the time dimension. An issue that emerges here is the interval of which the data should be collected and used, as there is currently no uniformity on how this should be done. Longer run versions of the regression bear significantly different results to the shorter ones. Fabozzi et al. (2014), explain that a two-pass regression technique is used to assess the model, which involves first an estimation of the Beta for each stock and then by using the ranked Betas estimated, portfolios are formed. At this point the regression analysis shifts from time-series to cross-sectional, which also hinders the model's ability to determine the over-time variation of the returns.

The Beta $\beta_{\alpha,t}$ in this case serves to denote the relationship between expected returns of an asset and its positive linear relationship to the systematic risk, with higher Betas signaling higher returns. This has also been found to not be accurate as several studies showed in later years, with Fama and French (1992), suggesting that lower returns can be associated with high Beta stocks and vice-versa.

Ross (1982), also, heavily criticized the fact that this type of analysis excluded a series of factors that could better explain the expected returns, than only estimating the Beta would. Moreover, Fabozzi et al. (2014), cite an earlier study conducted by Robert Jones, a quantitative analyst for Goldman Sachs, who found through multiple regressions that other factors, namely the market value, momentum and three separate risk factors (with systematic risk being one of them), add more validity to the expected returns' analysis. Multivariate analysis' results also hold for ex post testing (Cochrane, 2011), a problem that is also very common to linear regression models and the CAPM specifically.

3.5.5 Factor Analysis

As we have already discussed Multiple Linear Regression (MLR) is employed in an attempt to examine whether a linear relationship between an explained variable and a group of predictor variables exists. Factor analysis might appear analogous to it; however, the two methods exhibit a great deal of dissimilarities. As Fabozzi et al. (2014), suggest, one of the linear regression's assumptions is that all variables used are observable, with explained determinants being also random. Furthermore, independent variables are either random or deterministic in addition to being observable. The result of this assumption is that the dependent variables can be computed with an error margin, as they can be derived from the predicted values of the explanatory variables, which are the true values of them. Following that notion, the error contains no information whatsoever, as both it and the predictor variables are not correlated[38]. The errors of different regression equations can be correlated to each other however (or be cross-correlated as is the common terminology). Another characteristic of regression analysis is that not only are there no limitations on the number of observations one can use, but also regressions tend to bear better results with larger samples. This property of MLR analysis, is a trait also shared with factor analysis and a major one as it will be made clear in the following paragraphs.

One of the main goals of factor analysis is to determine whether the data composition can be displayed in a less complex form. Or to put it in a more simplistic way, whether it is possible to conduct a multiple regression analysis more effectively with a diminished number of variables. Factor analysis suggests that all the dependent variables y_i can be expressed as a number of unobserved variables called factors. The main assumptions of this methodology are first that the residuals and the error terms are zero-mean variables, and second that the correlation between them is zero. Multiple regressions on specific groups of variables (such as macroeconomic or company specific characteristics), can also be viewed as factor analysis.

[38] One of the fundamental assumptions in linear regression analysis is that $E(x|u) = 0$

3.5.6 Differences between Multiple Linear Regression Analysis and Factor Analysis

The differences between MLR and Factor analysis have already to an extent been explained, however it is important to address them further so that any confusion can be avoided. As already discussed, contrary to the deterministic variables of regression analysis, factors are unobserved variables. Moreover, the error terms in factor analysis are considered to be uncorrelated, while this statement does not hold for multiple regressions.

Furthermore, the fundamental equation of factor analysis[39], suggests that not all the independent variables contribute to the underlying factors. This assumption leads to the idea that for every group of factors there is always a matrix that contains orthogonal factors, namely those ones that are uncorrelated with each other and have unit variance. Specifically, for strict factor models[40] the correlation between variables is perfect and thus there are no residuals left. The result of this attribute of strict factor models is that more information (contained in observations) can be fitted in just one factor, and subsequently the size of the dataset is reduced while retaining all the relevant information of the original data. Moreover, strict factor models relax the assumptions of MLR analysis, mainly due to the elimination of the non-collinearity of the independent variables as well as the disassociation of said variables with the error terms and themselves.

3.5.7 Forms of factor models

As stated in Fabozzi et al. (2014), Tinsley Howard and Brown Steven (2014) and Gujarati (2015), factor models can be presented in a variety of forms, with the most prominent ones being:

[39] The fundamental equation is defined as: $Cov(F) = Cov(B) + Cov(u)$, where $Cov(F)$ is the covariance matrix of the underlying factors, $Cov(B) = BF\hat{B}$ is the covariance matrix of the factors and the coefficients and finally $Cov(u)$ is the covariance matrix of the error terms. For further explanation on this please see Appendix 2.3.

[40] Correlation of error terms in strict factor models is zero

1. The explicit form:

$$y_i = a_i + b_i f_i + \cdots + b_N f_N + u_N \tag{4.11}$$

where a represents the constant term, b the coefficients or factor loadings, f the factors (which are unobserved variables) and u the error terms.

2. The vector form:

$$y_{ij} = a_i + B_{ij} f_{ij} + \cdots + B_{Nj} f_{Nj} + u_N \tag{4.12}$$

where a indicates the vector of constant terms, B the vector of the factor loadings, f the vector of the factors and u the vector of the error terms.

3. Finally, the matrix form:

$$Y = F \times L + U \tag{4.13}$$

where Y stands for the matrix of the data, F is the matrix of the factors, L the matrix of the loadings and U the matrix of the errors. It is worth to note here that both F and U matrices are comprised of unobserved variables.

The final form of the matrices can be written also as: $x_{ij} = B f_{ij} + u_{ij}$, after we demean the data, which is done by simply subtracting the mean from each observation in the dataset. This process is of particular importance, since it allows us to transition from the original equation for the matrix form to a new one, namely $X = F\acute{B} + U$. This provides the opportunity to examine the independent variables of the data without having to determine the dependent part (which is this case is the discount rates for private companies).

3.5.8 Number of Factors, Parameters, Finite and Infinite Factor Models

Parameters on the other hand can easily be determined through Maximum Likelihood Estimation (MLE), which requires one additional assumption on the researcher's part. The independent variables have to follow a probability distribution that can be detected, for example the normal distribution. This methodology leads us to accurately determine the parameters of the factor models, with the additional constraint of the factors being uncorrelated as well as them having a variance equal to one.

On the other hand, factors are unobserved variables, and therefore accurately determining them is impossible. Instead of them, we compute scores called factor scores or predicted scores, Fabozzi et al. (2014), by using the parameters we have previously obtained, and afterwards using the independent variables as the dependent ones. The main assumptions of multiple regressions also hold in this case however, it should be noted that the scores do not exhibit the same properties as the factors themselves[41]. The factors are unobserved variables, they can be reconstructed from the data and are notional, which implies that it is difficult to explain what they really represent.

Another question that might arise at this point is what a valid number of factors might be. The answer is usually given by a criterion known as the Cattel scree plot (Fabozzi et al., 2014; Kim and Mueller, 1978), under which each asset in a portfolio is represented by an eigenvalue obtained from the factor analysis. A plot is then created, using these eigenvalues in a declining order. After a specific number of eigenvalues, the rate of deterioration for the values is more rapid and an elbow is created after which the rate decelerates. Other more common criteria are the Akaike and Bayesian ones (Jolliffe, 1986).

Factor models are especially useful in an infinite market setting. An infinite market is defined as one where the supply of the asset we want to acquire, trade on, etc. is unlimited. Unrealistic as this scenario might appear, it has proved to be very useful in the study of large markets, as their properties display similar properties to the infinite ones. In this setting the assumption of a diagonal matrix of errors is relaxed, since

[41] Not orthogonal and not variance equal to 1

this is what happens in real markets. This point however creates an extra set of factors that need to be estimated, as the errors now contain information on the original variables and can thusly be viewed as factors. Ross (1976) attempted to provide a solution to this problem with his Arbitrage Pricing Theory (APT), by incorporating an endless number of markets at an endless number of points in time to the existing factor model analysis.

The aforementioned methodology allowed researchers to develop the idea of two categories of factors within the models, the global and the local factors, which are respectively eigenvalues that either grow indefinitely or are limited. It is important to note that approximate factor models and the methodology which will be used in this thesis, namely Principal Component Analysis (PCA), are similar for large samples (but exhibit significant differences for smaller samples). According to Chamberlain and Rothschild (1983), the factors obtained through the process of the approximate factor models, are unique and can be obtained through the Principal Components Analysis, as the extraction of unique factors through this method coincides with the actual number of global factors in the model, however the main problem of accurately defining the number of these factors remains.

3.5.9 Principal Component Analysis

It is evident from the extensive literature on the topic of discount rates (regardless of the term used to define it, whether it is called risk premia, expected returns, etc.), that the great variety of theories covering the topic, gave birth to a long series of different variables. As already briefly mentioned, there are two main theoretical paths, with three subcategories each (Cochrane, 2011). Studies such as that of Cooper and Priestley (2009); Gospodinov et al. (2014); Graham and Harvey (2001); Møller and Rangvid (2015); Pereiro, (2001), focus on the investor's aspect of the discount rates, with macroeconomic theories on consumption, risk investment and equilibrium being the key factors examined. Others focus more on how investors behave (for example Cooper and Priestley, 2016; Cooper et al., 2005; Jiang, 2013; Krüger,

Landier, and Thesmar, 2015), while the most explored theoretical framework is that which deals with return-based factors and other financial characteristics. These theories appear to have settled within the relative literature and are therefore less actively researched at this point.

The second branch of the literature focuses on three key aspects. Firstly, market segmentations and how the investors in different market settings react to risk stimuli (Hwang, Lee, Lim, and Park, 2013; Maio, 2013). The next field of research is leverage and how it affects investment decisions, as it is represented in studies such as Choi and Richardson (2016) and Mandelker and Rhee (1984)). The last but perhaps one of the most important, the liquidity of an asset (Da et al., 2012; Pástor and Stambaugh, 2003). Liquidity is a term that involves not only how easy it is to transact an asset, but also how an asset behaves in times of distress (such as in the financial crisis of 2008) and whether this asset fills a specific trading gap. The second group of theories is the epicenter of an ongoing debate, as researchers try to determine the effect of these variables on the discount rates.

In a setting like this, where there is an abundance of potential factors, a unified framework that can consolidate them is needed. What is proposed with this thesis is the adoption of a methodology called Principal Component Analysis (PCA). This methodology was formulated over a century ago, however due to the complexity of the calculations needed, it has only recently become popular. The idea behind it is that it can decrease the dimensions of a dataset, that includes a great number of highly correlated variables (in this case the literature points on the degree of correlation between them), while simultaneously preserving the highest level of variation possible for the dataset.

As will be seen below, this is achieved by expressing the existing variables into a new set of variables, that are linear combinations of the previous ones. Moreover, the components created are uncorrelated and ordered in a decreasing way, with the first few of them containing most of the variation from the original dataset (Jolliffe, 1986). That also suggests that the components include all available information within them[42]. To avoid any confusion with the factor analysis described above, it is worth noting that principal

[42] This is also the reason why this methodology can be considered more versatile than regression analysis in determining the variables that matter.

component analysis has the significant advantage of not assuming any underlying model or structure for the data. Finally, principal components, as opposed to factors, are observable variables, but this is something that will be discussed extensively in a later section.

3.5.10. The historical development of the method

In a paper by Pearson (1901), a methodology was described about a group of lines that connected a number of points in a p-dimensional space. His idea was that the points in these lines were the ones that would be the best fit to any data and would lead potentially to what we have defined as Principal Components. What is also interesting is that Pearson was convinced that despite the difficulty of the calculations after the fourth component it would still be feasible to accurately use this method.

Another study by Hotelling (1933), explains that it is possible to express the original independent variables by a more concise group of variables, which he names components, so as to avoid using the "factor" word, which had different uses in other fields. The goal of these new variables is to contribute the maximum amount of variance to the total variation of the original ones. He then named these variables Principal Components. His methodology is similar to the one used to date, however there are three major differences. The first one is that he uses the correlation between variables instead of the covariance, which is more customary now. He also considers the original independent variables as the linear combinations of the components instead of vice versa. Lastly, he avoids presenting these variables with a matrix notation.

According to Jolliffe (1986) calculations were impossible until the introduction of personal computers. Combined with the expansion of the statistical research over the past three decades, various applications of PCA have been developed. This resulted in the methodology being used in a significant number of different scientific fields, ranging from psychology and meteorology, to finance and biology.

3.5.11. The mathematical and statistical properties of PCA

We start this analysis by considering the following: As seen from the literature review, a plethora of determinants arise as practitioners or academics describe the factors that affect the cost of capital of a company. These factors cover a broad spectrum, ranging from macroeconomic elements and company specific indicators, to less quantitative variables such as the information dissemination from the company or tax-related legislation. The main notion of PCA is to create linear combinations of the variables that are mutually orthogonal and have the utmost variance possible. The meaning of "orthogonal" in the PCA context is not the same as in factor analysis, as components found by this methodology do not have any structure whatsoever, or to put in an algebraic context their dot product[43] is equal to zero.

3.5.11.1 Eigenvectors and Eigenvalues

Before commencing the eigenvalues and the eigenvectors require definition, since those are key elements of the methodology. If there is a matrix $\boldsymbol{B}$ with *n x n* dimensions, any vector *y*, that has the property

$$\boldsymbol{By} = \lambda \boldsymbol{y} \tag{4.14}$$

for any constant number λ, to be called the eigenvector of $\boldsymbol{B}$. λ is the term of the eigenvalue of *y*, meaning that eigenvectors are the vectors that are converted by the matrix to another vector that is a linear transformation of the original one. Eigenvalues of the same eigenvectors are scalar products of each other. It is also interesting that if the original matrix $\boldsymbol{B}$ is symmetric then the eigenvectors produced by it are orthogonal, which suggests that the scalar product of eigenvectors of different eigenvalues is zero, and if the original matrix $\boldsymbol{B}$ is diagonal all the eigenvalues from it are the diagonal elements.

In order to properly define the link between the eigenvalues and eigenvectors, the start point can be seen whereby any symmetric matrix $\boldsymbol{B}$ can be written as:

[43] This can also be found as scalar product in the relative literature

$$\boldsymbol{B}y = \boldsymbol{y}\boldsymbol{\Lambda} \tag{4.15}$$

Since $\boldsymbol{B}$ is symmetric, *y* will be and *n x n* orthogonal matrix with columns being the eigenvectors of *B*, so that $y^{-1} = y'$. Moreover, $\boldsymbol{\Lambda}$ is the diagonal matrix of the eigenvalues. In this case the spectral decomposition of B becomes: $\boldsymbol{B} = y\boldsymbol{\Lambda}y'$, which is just a variant of the original *B* matrix.

The above can be transformed to:

$$(\boldsymbol{B} - \boldsymbol{\lambda I})\boldsymbol{y} = 0 \tag{4.16}$$

where *I* is the unit matrix[44]. Per matrix algebra if there is an invert to the the product $(\boldsymbol{B} - \boldsymbol{\lambda I})\boldsymbol{y}$, then *y* = 0. The result of this is that $\boldsymbol{B}$ has a zero determinant, which allows us to find the eigenvalues by calculating:

$$det(\boldsymbol{B} - \boldsymbol{\lambda I}) = \boldsymbol{0} \tag{4.17}$$

we know from basic linear algebra the equation expressed above has *n* number of solutions. If we solve this for λ we find a series of eigenvalues $(\lambda_1, \lambda_2, \lambda_3, \ldots . \lambda_n)$. For every one of them there is a unique eigenvector y_i, and by solving the original equation ($\boldsymbol{B}y=\lambda y$) we find a solution for any linear combination of λ with any constant.

3.5.11.2 The process of defining the Principal Components

After having defined eigenvalues and eigenvectors, and assuming a vector *x* of $\boldsymbol{r}$ number of variables[45]. This vector has a known covariance matrix denoted as $\boldsymbol{\Sigma}$[46]. This matrix's elements are the covariance between the *i*-th and *j-th* elements of the original matrix *x*, when $i \neq j$, and the variance when $i=j$. Following what we have seen in the previous paragraphs, we can say that there is a matrix $\boldsymbol{C}$ with the elements in its columns being the eigenvectors and another matrix $\boldsymbol{F}$ with the eigenvalues being on the main diagonal, as

[44] All elements besides the principal diagonal are 0 and the ones on the diagonal are 1.
[45] These variables are random.
[46] As per common mathematical notation for the covariance matrix.

those can be found by the equation: $\boldsymbol{\Sigma_r C_i} = \lambda \boldsymbol{C_i}$. Then the eigenvalues and eigenvectors are calculated following the procedure described above.

In addition, it is important to note how the different types of data included in this study affect the methodology. As Williams and Abdi (2010), explain, before applying PCA the data need to get processed so that each column of the data table consists of zero-mean elements. The input then becomes either the covariance or the correlation matrix, depending on whether the data are standardized to have zero mean and variance of one. We use the correlation instead of the covariance matrix in the case of different measure units in the data.

Following this the principal components are created. In order to do that the original dataset (***r***) is multiplied by the eigenvector matrix, to create the product: $\boldsymbol{P} = \boldsymbol{Cr}$, where ***P*** are the principal components expressed as the columns of the matrix of the product of the ***C*** eigenvalues' matrix and the variables' matrix ***r***. At this point it is possible to revert to the original data. This can be done by multiplying the components with the eigenvector matrix transpose namely:

$$\boldsymbol{r} = \boldsymbol{rCC'} = (\boldsymbol{rC})\boldsymbol{C'} = \boldsymbol{PC'} \tag{4.18}$$

The *i*th element of the new data matrix is a weighted sum of the principal components, with the weights of each component' *j*th being the *i*th element of the *j*th eigenvector. The spectral decomposition method is used on the covariance (correlation matrix) ***C*** so that:

$$\boldsymbol{C} = \boldsymbol{WFW'} \tag{4.19}$$

where the matrix **W** represents the factor weights. Also, in the case of the eigenvector C having unit length then the variance of each component can be given by: $var(P)=\boldsymbol{F}=\lambda_i$. Finally, if ***C*** is the correlation matrix the sum of its eigenvalues coincides with the number of variables in the original dataset.

At this point those components that represent the maximum variance within the sample are located. The components are ordered according to the variation they explain. The eigenvalues order is done from the

most to the least substantial in terms of its magnitude. The proportion of the total variation explained can be found simply by dividing the eigenvalue of its component to the total of the eigenvalues.

If the variables in the dataset are highly correlated, then with only a small number of them, the whole dataset can be represented (Fabozzi et al., 2014; Jolliffe, 1986). Specifically, since by design, the first few components account for most of the variation of the variables, it is easy to express those as a linear combination of the first few components:

$$\boldsymbol{r = W_{i,1}P_1 + W_{i,2}P_2 + \cdots + W_{i,n}P_n} \tag{4.20}$$

This has a lot of significance as will be seen at a later stage in the analysis. The final point that needs to be stressed here is that the components with the largest variance, also exhibit another very interesting characteristic. They minimize the SSR[47] and therefore their weights act as the $\hat{b}$ in linear regression analysis. The process of performing the PCA can be summarized in the words of Fabozzi et al. (2014), p. 259:

> "A covariance (or correlation) matrix of the data needs to be determined first, from which are extracted the eigenvectors and eigenvalues of the matrix. Afterwards, these eigenvectors are multiplied by the elements of the original dataset, and from the product of this process the principal components are estimated. Subsequently, they are ranked, based on the magnitude of their contribution to the total variation of the sample, the first few with the highest eigenvalues are chosen to represent the data as weighted sums of the components".

The question that rises from the last part of the methodology is how many of those components are enough to lead to valid conclusions, and it is still a subject of debate throughout the multivariate analysis literature. For instance, Bai and Ng (2002), argue that although one of the most important aspects of factor analysis is an accurate assessment of the number of factors, researchers use arbitrary criteria, as most of the times factors are decided by the data. For that reason the authors propose a methodology that allows the selection of the determinants through the constraints of time and sample size. Jolliffe (1986), reviews all available

[47] Sum Square of Residuals

methodologies, and suggests that the most commonly used in practice are those techniques that can be seen as rule of the thumb techniques, where the components are simply chosen based on criteria, such as the total percentage of variation explained, the size of the variation explained by individual components and the scree plot. The author explains that these methods work in practice, an idea shared by a significant part of the literature (Ahn and Horenstein, 2013; Fabozzi et al., 2014), with some variations.

3.5.11.3 Differences between Factor Analysis and PCA

After the discursive analysis, conducted on both methodologies, factor analysis and PCA may seem similar. However, it is noted that these methodologies have major differences. The first and most important one is that although both methodologies employ a covariance matrix as their focal point, they focus on different aspects of the matrix. PCA focuses on the diagonal elements of it, while factor analysis on the off-diagonal ones. As Chamberlain and Rothschild (1982) explained however, principal components perform exquisitely when interpreting the off-diagonal elements of the matrix, since by design they account for most of the variability of the data. This means that the components can be a preliminary solution to the factors of factor models.

Moreover, another difference is that Principal Components can be used to accurately recreate the original dataset. This characteristic of the components can be attributed to the fact that they are linear combination of the original variables. Factors, however, are not. In the case that the i.i.d. assumption is made about the data, factors are linear combinations of the data with an error and that might be the closest the factors come to the principal components. Even though that is not always the case, as a linear relationship can be only one of the underlying structures of the variables.

Another difference is that independency among the variables is always exhibited by a principal component. Specifically, Jolliffe (1986) explains that any variable that is independent from the others will receive its own principal component, and it will bear a significant amount of information regarding the variability of the original dataset. In order for a factor to be created, at least two variables in the set need to contribute to

it. This means that a single variable will not stand out in the analysis and will be subsequently moved to the error part of the analysis. The significance of this observation is that the factors included in a factor analysis are always less than those used in a PCA, if both are applied on the same dataset and attempt to account for the sample's variability.

3.5.11.4 The proposed models of the thesis

The main idea of this thesis is to consolidate all the factors that affect the discount rate, as they have been identified in the literature, under a common framework. The PCA analysis will be applied to public companies' data, to distinguish those determinants, which affect the variation in the discount rate the most. The original model is:

$$DR_i = a_i + \beta_i X_{i,j} + e_{i,j} \tag{4.21}$$

Where: DR_i is the matrix of the discount rates[48],

a_i is the vector of the constant;

β_i is the vector of the coefficients;

$X_{i,j}$ is the matrix of the variables identified in the theoretical framework;

$e_{i,j}$ is the matrix of the residuals;

After we apply PCA we will be able to express the variables in a more concise way:

$$X_i = W_{i,1}P_1 + W_{i,2}P_2 + W_{i,3}P_3 + \cdots + W_{i,n}P_n \tag{4.22}$$

[48] The PCA does not require us to have the data for this matrix, however the Discount Rate variable is stated here for statistical purposes.

Where each new variable X will be a linear combination of a number of weights and the original variables. This will reduce the size of the dataset and the most important variables will be highlighted as those will account for most of the variation.

Following the methodology of Boone et al. (2007), Callahan et al. (2003) and Madrid-Guijarro et al. (2009) we will use these variables to create an index. Given the fact that the original dataset consists of panel data, as we want to capture the evolution of the discount rates over the sample period, and because as Boone et al. (2007) and Gujarati (2015) explain, we will include industry fixed effects in the regression models, as companies within the same industries operate under the same market conditions, as the previously cited sources suggest. Fixed effects are used when the researcher wants to analyze the impact of variables that vary over a specific period. Fixed effect methods are capable of isolating and removing attributes that may impact or bias the independent variables. Moreover, they account for the difference in the characteristics of the firms within the sample. The model that we will use is the following:

$$Y_i = a_i + B_{1i}X_{1i} + B_{2i}X_{2i} + e_{i,j} \quad (4.23)$$

Where: Y_i is the ratio chosen to represent the discount rate in the thesis

a_i is the vector of the constant;

B_{1i} is the vector of the coefficients for the control variables;

B_{2i} is the vector of the coefficients for the components, established through the previous test methodology;

$X_{i,j}$ is the matrix of the variables identified by the PCA methodology, together with the macroeconomic variables from the original dataset. The macroeconomic variables will act as control variables.

$e_{i,j}$ is the matrix of the residuals;

The reasoning behind splitting the variables into two distinct groups, one that refers to macroeconomic variables and a second that is dedicated to company-oriented ones[49], is best explained in the studies of Kaserer and Kraft (2003) and Boone et al. (2007), and is twofold. Firstly, the components created through PCA, are linear combinations of the original variables. To give them a proper economic interpretation, we need for the underlying variables with similar characteristics, which the components are based on, to be examined together, as similar traits will allow the identification of what the new variables represent. Furthermore, macroeconomic variables affect all companies, regardless of the industry they operate into, and so their effect will be present on all the companies in the dataset. In that sense it will be best if we can isolate the effects of each of those groups and study them separately.

Finally, as a means of testing the results, we will apply the results obtained from the original analysis, on the secondary dataset obtained for private companies from both the UK and the US. The final model, similarly to the previous one, but with the major difference that following the cross-sectional nature of the data in this one, we will use a multiple linear regression model, to estimate the effect of the variable identified by the previous model on the discount rate of private enterprises:

$$DR_i = a_i + \beta_i X_{i,j} + e_{i,j} \tag{4.24}$$

Where: DR_i is the ratio chosen to represent the discount rates[50],

a_i is the vector of the constants;

β_i is the vector of the coefficients[51];

$X_{i,j}$ is the matrix of the variables identified by the previous methodology (PCA);

$e_{i,j}$ is the matrix of the residuals;

[49] This would also include the variables that represent legislation related to specific company variables.
[50] The PCA does not require us to have the data for this matrix, however the Discount Rate variable is stated here for statistical purposes.
[51] Which includes macroeconomic and company specific variables, as those are defined previously by the PCA.

Despite the flaws of linear regression analysis, in this occasion it can be safely employed for mainly two reasons. First, as explained, the focus of the final part of the analysis is on the private companies. However, the only moments that private enterprises have a valuation ratio available is either during an M & A or prior to an IPO. Both these events are unique as they occur rarely, in the lifetime of a company. As such the data obtained are cross-sectional. The most commonly used methodology, as established above, in contemporary research, is linear regression analysis. In this case however, as we have included a great number of variables, multicollinearity (which will be extensively explained in the following section), might cause the estimators to become biased. As we are using the results from the PCA however, we have significantly reduced the chances of that happening. For those reasons, we are able to use multiple linear regression analysis.

3.5.11.5 Regression Issues Around Multicollinearity

In multiple regression analysis, one of the most commonly occurring problem is that of multicollinearity, especially in sets with highly correlated variables. Multicollinearity is observed when an independent variable can be expressed as a linear combination of another variable or in general is highly correlated with another variable, and it can affect the unbiasedness of an estimator of the regression, by increasing the overall variance of the sample in a considerable manner. One of the most common ways of dealing with it, is by omitting a regressor[52], which of course may lead to excluding a significant part of the explanatory analysis.

This is a difficulty that could arise in this study, since the data used consist of a number of highly correlated variables. One solution proposed for this, is to use PCA's components as the explanatory variables in the place of the original ones. As discussed the components are orthogonal[53], so there are no multicollinearities between them (Jolliffe, 1986). If some of the components are excepted, the model does not suffer from

[52] After having tested each variable separately and determining whether its sign and value are realistic.
[53] Principal components are uncorrelated to each other.

increased variances. As the first few components account for most of the variability in the sample and therefore they can be used without the prospect of experiencing multicollinearity problems.

Several variations of the PCA have been introduced in order to deal with the aforementioned situation. One of them allows deletion of specific parts of the components themselves. Another, named latent root regression, allows researchers to find components on both sides of the equation (both on dependent and independent variables). The new constructs created do not suffer from the problems of the original ones. Regardless of the variation used the result of countering multicollinearity remains.

It is important to note at this point, PCA is useful not only for dealing with multicollinearity problems in the regression analysis, but also for another reason. The components, estimated by this method, are an assortment of the original variables. The causality relationships described by multiple regressions can be more easily explained under the prism of the components' composition, as they are orthogonal relationships between the variables, and thusly can be more easily interpreted.

3.6 Concluding Remarks

To conclude the methodology, and before moving to the results section, it is necessary, as we covered several topics, to provide a comprehensive summary on the ideas developed previously and how this thesis is linked to them. As we have noted, the primary aim is to unify the literature and highlight the most prevalent factors impacting upon the constituent structure of the discount rate, in an attempt to create a framework with which these determinants will be used to achieve the most effective and efficient valuation outcome possible. To achieve that, the first step was to comb through the literature and ask a series of questions that will be the epicenter of the analysis. To answer these questions however we had to use the knowledge gained from the literature and build a dataset that encapsulate as many ideas as possible. This however creates a series of issues, with the most probable one being for variables to overlap and thusly affecting the final outcome of the analysis. To counter the effect of multicollinearity, as this overlapping is called, we employ a methodology that was developed in the beginning of the twentieth century, however

its implementation became possible with the increase in computing power, namely Principal Component Analysis (PCA).

PCA allows us to create an index, which is essentially a "summary" (Callahan et al., 2003) of the original variables. The new variables, that constitute the index, are linear combinations of the original determinants, which retain most of the variation of the original dataset. The components (as the new variables are called) are ranked based on the percentage of variation they explain, with the first component explaining the most, and the variance explained diminishing as we progress through the lower ranked variables. The selection of the appropriate number of components is an issue that we will refer to extensively in the Results and Analysis discussion chapter that follows. The index created, will be used as the explanatory variable for the variables (together with a number of macroeconomic variables that act as control variables), for the regression procedure against a valuation multiple, which will be the P/E ratio, as shown in the paper of Lee and Masulis (2011) (this paper is one in a series of papers that deal with the process of companies going public, and what ratios are used by analysts for their pre-IPO valuation).

As the primary objective of this study is to analyze what affects the discount rate in the valuation of private companies, we will use the results from the PCA, regress them on the multiple, and use the components that will be created through this methodology on a dataset that consists of UK and US private companies.

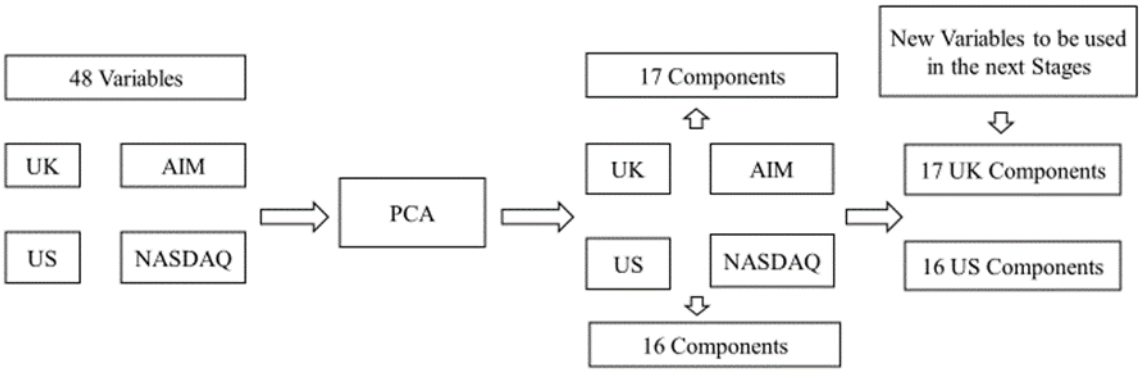
Having established the components through PCA, and having conducted the panel regression analysis, we will proceed to formulate our methodology for the private companies that we want to examine. As the data we will be using will be provided through private companies' M&A transactions, we will be using Multiple Linear Analysis, as the nature of our data is for them to be cross-sectional. For this reason, we need to determine what will constitute our dependent and independent variables. To accomplish this task, we turn to the related literature. The first study that we will use as a compass, is that of Officer (2007). To determine the illiquidity discount, that is ever-present in private companies' transactions, Officer compares the value of private M&A deals, to those of equivalent (in terms of value, company size, industry and time of the deal) public ones. Similarly, to him, we will also use a separate dataset of public M&A deals, that we will

match using the same criteria, to compare with our private companies' M&A deals, so as to determine the illiquidity discount. For the independent variables, we will simply match the private companies to those featured in our PCA components, following the same pairing criteria (industry, size and time) as in the relevant literature (see for example Asker et. al, 2015), and subsequently use the components from those companies identified as proxies to the private companies under investigation. (A visual display of the aforementioned methodological procedures can be found in the section of the Appendix, titled: Methodology Roadmap)

According to prior literature, we expect these results to be transferrable from the public to private companies (as the comparable method is the most commonly applied valuation approach). Should those results hold for private enterprises as well, then it will be an indication that there are no significant differences in the characteristics between public and private comparable companies and appraisers and academics can use the final results to assist in determining an appropriate discount rate. Finally, as we are contrasting companies that operate in different countries, we will be able to extract information on how companies behave under a different legislative framework. The results being consistent on these markets will attest to the models' ability to accurately predict the discount rates' variability in a global setting and provide us with an opportunity to observe differences between UK and US investors. Figure 8, below, provides a methodological roadmap for the analysis and the data used in this thesis.

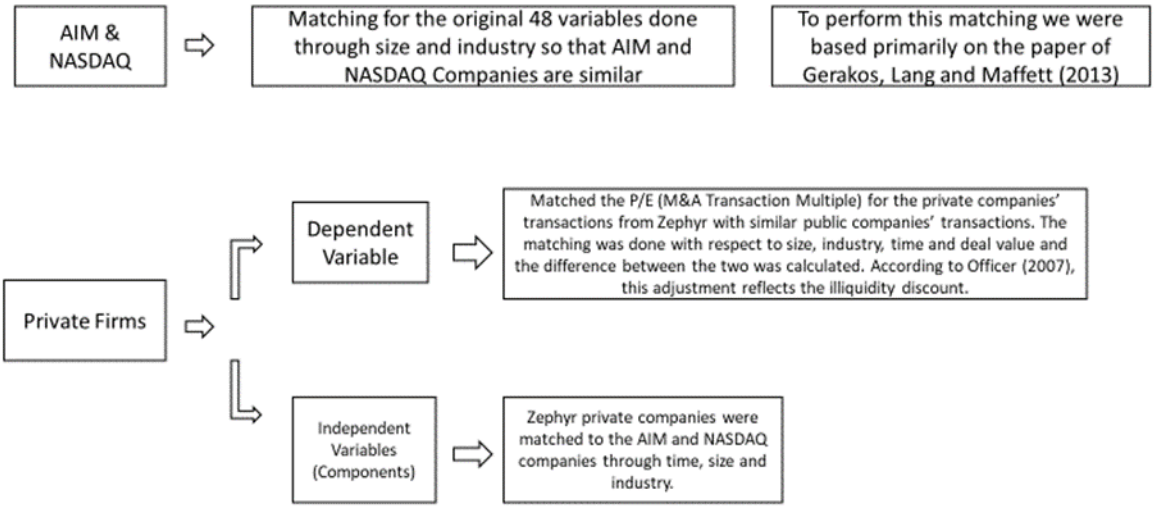
Methodology Roadmap

Stage 1: PCA



Methodology Roadmap

Stage 2: Sample Matching



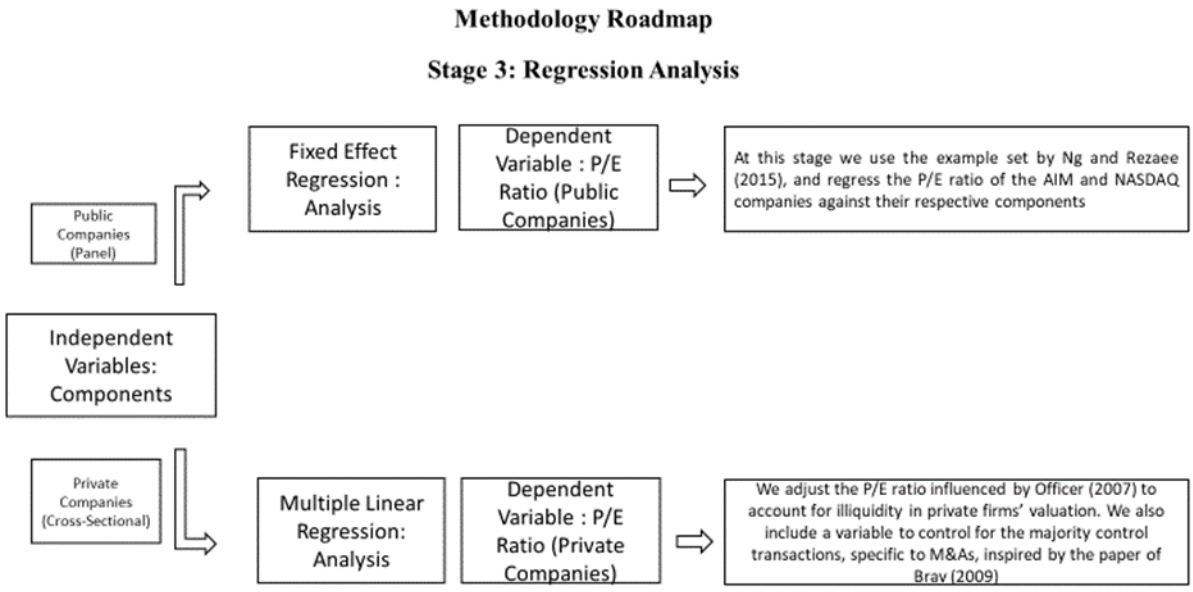


Figure 8: Methodology Roadmap

4. Empirical Results

In this section we will be discussing and explaining the analysis and by doing so we will provide the answers to the research questions, as those were presented in chapter 3. In order for to do that, we will be following the methodological ideas proposed in several earlier papers both in terms of how the methodology should be applied (with some of the key studies on that notion being Boone et al., 2007; Callahan et al., 2003; Kaserer and Kraft, 2003; Madrid-Guijarro et al., 2009; Ng and Rezaee, 2015), but also on how the sample for the private companies can be constructed and applied (some of the most important papers in that regard are those of Brav, 2009; Michaely and Roberts, 2012; Officer, 2007).

We will begin the analysis by demonstrating the multicollinearity issues that are most certain to arise in a complex and vast dataset such as the one used in this thesis. In order to do that we will perform a simple linear regression of the variables of interest on the valuation multiple (P/E), we chose as the proxy for value. Following that we will perform a Variance Inflator Factors test and present the results. As a robustness check we will also conduct a stepwise OLS regression, to examine whether some variables are deemed redundant, due to multicollinearity concerns. We will then proceed with the PCA on both the UK and US samples, because as discussed in the previous chapter it will allow us to counter the effects of multicollinearity. The components that will arise from this process will be discussed and analyzed, so as to understand what each of them represents, and then those will be used as independent variables in panel regressions, to determine the explanatory power of each one of them.

The next step in the analysis is to provide a detailed explanation as to how the private companies sample was constructed. This will include, but not be limited to, explaining how the matching between the AIM and NASDAQ companies was done and on which were the key studies that served as an example in this endeavor (for instance studies such as that of Abudy et al., 2016), in the sense that they engaged in similar topics as the one in this thesis. The data for the private companies will be dictated by the panel regression results of the AIM and NASDAQ and will serve in the final part of the analysis, as regressors in a multiple linear regression with the valuation multiple. This process will provide us with inference on how these

variables explain the valuation of private firms, by allowing us to back test the original results from the panel regression analysis.

The chapter will begin, as discussed, by examining the effect that the selected variables have on the valuation of the companies of the sample. As a measure of valuation, the P/E ratio is applied (following the log-linear specification explained previously and as suggested by (Elnathan et al., 2010) as this is defined in the study of Lee and Masulis (2011)). The price to earnings ratio being a relative valuation ratio incorporates three key drivers of value namely the cost of capital, the earnings growth rate, and the payout ratio. This will serve as a proxy measure for the discount rate. A multiple regression analysis is employed, after controlling for the important macroeconomic characteristics, as those are identified in the related literature, previously discussed. This regression, however, serves no other purpose other than allowing us to use it, to examine potential multicollinearity issues at a later stage, through the variables' VIF and Tolerance. The results for both the UK and the US are reported in Table 15 that follows, they cannot be used for inference purposes, however, as the regression is simply run to indicate potential multicollinearity between the variables:

	UK	**US**
Variable	**Panel A**	**Panel B**
Financial Crisis	12.01	**-47.15***
	(0.43)	(-1.65)
Volatility	**0.49*****	**-4.71***
	(3.25)	(-1.86)
IAS 34/Jobs Act	-7.06	**-92.02*****
	(-0.36)	(-3.23)
Comp. Act /FAS 123R	**45.45****	48.70
	(2.78)	(0.51)
Entry Amendments	45.13	-35.77
	(1.15)	(-1.36)
Effective Tax Rate	**-3.91*****	**-1.00****
	(-3.15)	(-2.69)
Big 4 Auditors	-15.94	-35.63
	(-0.63)	(-1.45)
Intangible Assets	**-0.22****	**23.83*****

	(-2.23)	(4.17)
Tobin's Q	**4.28*** **	**-50.16*** **
	(3.59)	(-3.51)
Risk Management Practices	**-38.40** **	-47.47
	(-2.45)	(-0.17)
Assets	**49.58*** **	**48.67** **
	(4.40)	(2.52)
EBITDA	**-0.03*** **	**-0.64** **
	(-5.19)	(-2.72)
R - Square	-11.02	59.82
	(-0.50)	(0.73)
ROE	-0.13	-0.31
	(-0.31)	(-0.37)
ROC	**3.98** **	-0.15
	(2.83)	(-0.77)
Retention Ratio	**-1.89** **	**46.01*** **
	(-2.68)	(8.28)
Depreciation	**1.78*** **	**0.95** **
	(3.40)	(2.63)
Earnings Yield	0.01	0.10
	(0.69)	(0.63)
Debt to Equity	**0.21** **	-0.01
	(2.64)	(-1.39)
Enterprise Value	0.02	**0.49*** **
	(1.09)	(9.61)
Insiders Stock	**-1.70** **	0.01
	(-2.14)	(1.28)
Board Composition	-10.77	66.33
	(-1.08)	(0.63)
Compensation to Assets	-0.03	**0.68*** **
	(-0.88)	(3.06)
No. of Insiders Holding Stock	**21.14***	**15.83** **
	(1.96)	(2.49)
Inventory to Sales	**0.01*** **	-0.03
	(4.48)	(-0.83)
Net operating Margin	0.001	0.001
	(0.52)	(1.18)
Dividend Growth	1.70	0.03
	(0.28)	(0.04)
FCFF	-0.01	-0.09
	(-0.01)	(-1.27)
FCFE	**-0.35*** **	-0.11
	(-3.76)	(-1.74)

Capital Expenditure	**0.35*** **	0.20
	(3.06)	(1.05)
WACC	**13.85** **	58.14
	(2.37)	(1.59)
Cost of Equity	**-10.25** **	**-72.01** **
	(-2.03)	(-2.05)
Cost of Debt	**5.39***	**-26.47*** **
	(1.78)	(-3.26)
Z-Score	**-0.05***	**-0.12** **
	(-1.76)	(-2.84)
MPK (Industry)	0.00	-0.00
	(0.77)	(-0.67)
ROA (Industry)	**-0.01*** **	**-1.71*** **
	(-4.2)	(-3.07)
IK (Industry)	0.32	-0.01
	(0.8)	(-1.31)
Alpha	0.03	0.11
	(0.42)	(0.45)
Beta	-13.67	-20.76
	(-1.53)	(-1.21)
Total Beta	0.00	-0.01
	(1.32)	(-0.1)
Constant	**-137.22***	**-422.07** **
	(-1.65)	(-2.63)
Adj. R-Square	**0.014**	**0.070**
F- Stat	**12.18**	**17.62**
p-value	**0.00**	**0.00**

Table 15: Multiple Linear Regression on the original variables and the valuation multiple (P/E). The significance on the variables is denoted by the number of stars next to the coefficients, with p = * for 10%, ** for 5%, and * for 1% significance. The t-statistics are reported in the parentheses below. This regression is performed to highlight potential multicollinearity issues, as it will serve as the base for the VIF tests, that will be performed in the following section, where we address the multicollinearity concerns and not for statistical inference.**

Panel A reports the results for the United Kingdom. As can be seen, the results indicate that a majority of variables are not statistically significant, with ROA, FCFE, Capital Expenditure, Inventory to Sales, Depreciation, EBITDA, Assets, Tobin's Q, Effective Tax Rate and Volatility, being significant at a 1% level and Retention Ratio and Insider's Stock at the 5% level. We also have Companies Act 2006, Intangible Assets, Risk Management Practices, ROC, Debt to Equity, WACC and Cost of Equity exhibiting significance at a 5% level. Furthermore, the model is characterized by a striking lack of explanatory power,

as this is displayed by the low adjusted R-Square (0.014), which can also be attributed to the fact that multiple regression models have their adjusted coefficient of determination "taxed" with the inclusion of each new variable[54].

As we can see in Panel B, results do not improve significantly for the US regression model, as it is also plagued by low t-statistics on the explanatory variables. Specifically, the Jobs Act, Intangible Assets, Tobin's Q, EBITDA, Retention Ratio, EV, Compensation to Assets, Cost of Debt and the industry ROA, are statistically significant at the 1%, with a few others reflecting significance at the 5% (Assets for instance) and 10% levels. Moreover, the model displays poor explanatory capabilities, as the coefficient of determination is as noted in the UK model below 10% (0.07).

All of the above, together with the problem of misspecification (including too many variables and thereby artificially increasing the explanatory variable of the model), raise the questions of whether the results are accurate, what causes them to perform so poorly and what can be done to improve them. The usual suspect that causes these series of problems, in models that include a large number of variables, is multicollinearity. In the following section we will address this issue.

4.1 Linear Regression Model and Multicollinearity Issues

In the previous chapter the impact of potential multicollinearity issues on Multiple Linear Regression was noted. As highlighted, multicollinearity violates the assumption of the Linear Regression Analysis, which requires that the independent variables within the model should not be linear combinations of each other. Although a linear relationship among the explanatory variables may be natural as Gujarati and Porter (2009) explain, it might create a series of problems regarding the explanatory capabilities of the model, regardless

[54] As we have a total number of 40 explanatory variables, it is expected that the Adjusted R-Squared would be low.

of whether the linear dependence is absolute (perfect multicollinearity) or limited (imperfect multicollinearity).

Several reasons as to why this phenomenon appears, have been identified, ranging from misspecification of the model employed to the inclusion of an abundance of variables with an insufficient number of observations. Moreover, models such as the one used in this thesis, which try to explore the relationship of many variables simultaneously, are more prone to have variables that may be linear combinations of each other. What is of interest however is the disruptive effects this has on the predictive abilities of the linear model used. Specifically, high multicollinearity is associated with high variances and covariances within the OLS estimators, which in turn leads to the wrong variables being deemed significant or insignificant, and this results in the null hypothesis being, wrongfully, more likely to be accepted. In addition, and as a consequence of what was mentioned previously, multicollinearity issues are associated with increased sensitivity for the linear regression estimators and their standard errors, even when the changes in the dataset are minimal.

Given the critical sensitivity described, the detection of multicollinearity within the model is imperative. There are numerous ways that this problem can be diagnosed. A first indicator can be the high correlations that can be observed between variables. Furthermore, an increased coefficient of determination paired with insignificant coefficients for the variables are also an indicator. One other way to identify it, which is the approach adopted in this study, is by examining the rate at which the variance and covariance of the regression coefficients increase. This can be achieved with a Variance-Inflating Factor test (VIF), defined as: VIF = 1 / (1-R^2), where R^2 is the coefficient of determination of regressing each explanatory variable against all the other explanatory variables. As Gujarati and Porter, 2009 indicate, VIF tests allow us to determine how multicollinearity artificially boosts the variance of the coefficients[55]. The inversed VIF is called tolerance and can be used instead of VIF but gives equivalent results. VIF values above 10 are

[55] Specifically, they mention that as we approach the value of 1 for VIF, the model becomes increasingly less collinear.

considered highly collinear, while multiple sources within the literature (Jolliffe, 1986; Wooldridge, 2002; Gujarati and Porter, 2009) mention that the minimum threshold for it should be set at 5, with others even being stricter and reducing the threshold to a VIF value of 3. Similarly, a value of tolerance approaching zero is also an indicator of multicollinearity.

Variance Inflation Factors

UK			USA		
Variable	**VIF**	**Tolerance**	**Variable**	**VIF**	**Tolerance**
Capital Expenditure	26460.44	0.000038	Compensation to Assets	1723.56	0.000058
FCFE	24255.92	0.000041	ROA (Industry)	1567.64	0.000638
FCFF	6619.82	0.000151	Z-Score	14.65	0.068261
WACC	15.67	0.063833	EBITDA	8.02	0.124627
Cost of Equity	15.60	0.064109	Depreciation	7.50	0.133406
Depreciation	3.50	0.285530	Capital Expenditure	6.77	0.147676
Intangible Assets	3.46	0.288783	Cost of Equity	4.91	0.203714
Companies Act	2.18	0.459058	WACC	4.87	0.205143
Financial Crisis	1.89	0.529791	Financial Crisis	3.88	0.257683
IAS Framework 2005	1.84	0.542364	Entry Amendments	2.61	0.383222
No. Insiders Holding Shares	1.52	0.657285	FAS 123R	2.50	0.400693
IAS 34 2014	1.45	0.688736	Jobs Act	1.75	0.571811
ROE	1.43	0.699848	Assets	1.70	0.588436
Debt to Equity	1.28	0.780311	Big 4 Auditors	1.55	0.645327
Risk Management Practices	1.26	0.791466	FCFF	1.33	0.751908
Board Composition	1.25	0.800635	Intangible Assets	1.30	0.767951
ROC	1.15	0.868816	Volatility	1.29	0.772202
Tobin's q	1.14	0.875391	FCFE	1.24	0.803562
Cost of Debt	1.13	0.885441	Enterprise Value	1.24	0.805905
EBITDA	1.13	0.885713	Cost of Debt	1.22	0.819509
Retention Ratio	1.10	0.911178	Tobin's q	1.19	0.840082
Volatility	1.08	0.923816	Risk Management Practices	1.11	0.898387
Effective Tax Rate	1.07	0.930645	ROE	1.08	0.928508
R-Square	1.06	0.940890	Net Operating Margin	1.07	0.932991
Dividend Growth	1.05	0.949916	Inventory to Sales	1.07	0.934178
Enterprise Value	1.04	0.958984	R Square	1.07	0.938861
Total Beta	1.04	0.965093	Retention Ratio	1.05	0.954221
Assets	1.02	0.982358	Earnings Yield	1.03	0.967346
Net Operating Income	1.02	0.983029	Total Beta	1.03	0.969222

Insider Holdings	1.02	0.983529	No. Insiders Holding Shares	1.02	0.978601
Inventory to Sales	1.02	0.984131	IK (Industry)	1.02	0.983949
IK (Industry)	1.02	0.984398	ROC	1.01	0.986933
Beta	1.01	0.988098	Dividend Growth	1.01	0.991761
Big 4 Auditors	1.01	0.989679	Board Composition	1.00	0.995130
Z-Score	1.01	0.991318	Debt to Equity	1.00	0.995866
ROA (Industry)	1.00	0.995811	Insider Holdings	1.00	0.998748
Alpha	1.00	0.998461	Effective Tax Rate	1.00	0.999601
Earnings Yield	1.00	0.998464	Beta	1.00	0.999680
Compensation to Assets	1.00	0.999297	MPK (Industry)	1.00	0.999755
MPK (Industry)	1.00	0.999523	Alpha	1.00	0.999802
Mean VIF	1435.34			84.51	

Table 16: Variance Inflation Factors (VIF) and Tolerance for the variables used in the thesis

As can be seen from Table 16, both the UK and US multiple linear regression models, suffer from a significant level of multicollinearity, as indicated by the high Mean VIF indicators (1435.34 for the UK and 84.51 for the US). In the case of the United Kingdom, the main contributing factors are seven variables: Capital Expenditure (26460.44), both FCFE and FCFF (with 24255.92 and 6619.82 respectively), WACC (15.67) and Cost of Equity (15.6). Borderline variables are Depreciation (3.5) and Intangible Assets (3.46). For the United States more variables seem to be significantly collinear. The nine primary variables are, Compensation to Assets which is the leading variance inflator factor with a VIF of 1723.56, followed closely by ROA (1567.64), Altman's Z-Score (14.65), EBITDA (8.02), Depreciation (7.5), Capital Expenditure (6.77), Cost of Equity (4.91) WACC (4.87) and the Financial Crisis (3.88). What is also noteworthy, is the diversity among the factors in the two datasets, with only Depreciation, Capital Expenditure and Cost of Equity overlapping on each test.

As we want to further exemplify the issue of multicollinearity, we will also perform a stepwise OLS[56], in addition to the MLR, and the subsequent VIF analysis, we have performed. This methodology can be used to effectively "evaluate" whether an independent variable should be added or removed from an OLS

[56] We also considered other options, such as for example Partial Least Square Regression Analysis, which is akin to the PCA methodology, or even more so to Factor analysis. It is a technique that can be used to reveal a factor structure, not only in the independent but also within the dependent variables.

regression and how this will affect the results. In our case we will use this the forward selection aspect of this method on both the UK and US samples, by which we add new variables gradually, in an attempt to determine how this affects the goodness of fit to our model. This process stops when new variables that are being added do not statistically "improve" our model. The results can be found in the table that follows (Table 17):

	UK		**US**
Cost of Equity	-0.238***	Total Beta	-0.0185***
	(-0.0447)		(-0.00645)
Inventory to Sales	0.0389***	Inflation	-0.0675***
	(-0.00909)		(-0.00997)
WACC	0.271***	IK (Industry)	0.107***
	(-0.0459)		-0.00608
Inflation	0.0656*	Financial Crisis	-0.0249**
	(-0.0359)		(-0.0103)
Financial Crisis	-0.0794***	Volatility	-0.237***
	(-0.0285)		(-0.00662)
Volatility	-0.0720***	Jobs Act	-0.0426***
	(-0.0103)		(-0.0143)
IAS 34	-0.111***	FAS 123R	0.0456***
	(-0.0189)		(-0.0135)
Companies Act 2006	-0.0314**	Entry Amendments	-0.329***
	(-0.016)		(-0.0299)
No. of Insiders Holding Stock	-0.449***	Effective Tax Rate	0.0111**
	(-0.0128)		(-0.00432)
Effective Tax Rate	-0.0525***	Cost of Debt	0.0953***
	(-0.0103)		(-0.00635)
Big4 Auditors	0.0224**	Intangible	0.125***
	(-0.0103)		(-0.00554)
Total Beta	-0.0365***	Tobin's Q	-0.0220***
	(-0.0104)		(-0.00501)
Tobin's Q	0.0253**	MPK (Industry)	-4.540***
	(-0.0104)		(-0.00493)
Board Composition	0.0347***	Assets	0.479***
	(-0.0119)		(-0.0126)
Assets	-1.097***	EBITDA	-0.166***
	(-0.0124)		(-0.00729)
EBITDA	-0.0334***	R-Square	0.0347***
	(-0.00853)		(-0.00651)

Z-Score	0.0355***	ROE	-0.0981***
	(-0.00859)		(-0.00881)
IK (Industry)	0.0538***	ROC	-0.0573***
	(-0.0104)		(-0.00589)
ROC	0.0344***	Retention Ratio	0.0951***
	(-0.0101)		(-0.00408)
Cost of Debt	-0.0248**	Depreciation	0.0891***
	(-0.0109)		(-0.00729)
Insiders Stock	0.0473***	Earnings Yield	0.0879***
	(-0.0104)		(-0.00727)
Dividend Growth	0.0170*	Debt to Equity	-0.703***
	(-0.00973)		(-0.0085)
Debt to equity	0.0449***	Enterprise Value	0.359***
	(-0.0116)		(-0.00857)
Enterprise Value	0.0972***	Insiders Stock	-0.0828***
	(-0.0149)		(-0.00909)
Constant	1.491***	Board Composition	-0.0108**
	(-0.0109)		(-0.00455)
		Compensation to Assets	-0.126***
			(-0.00789)
		No. of Insiders Holding Stock	0.00902**
			(-0.00459)
		Inventory to Sales	-0.0577***
			(-0.00552)
		Net Operating Margin	-0.287***
			(-0.00749)
		Z-Score	0.0258***
			(-0.00817)
		FCFF	-0.207***
			(-0.00563)
		FCFE	0.105***
			(-0.0059)
		ROA (Industry)	-0.280***
			(-0.00741)
		WACC	-0.232***
			(-0.0225)
		Cost of Equity	0.247***
			(-0.0205)
		Constant	5.531***
			(-0.00595)
N	56.830		89.436

Table 17: This table presents the results from the Stepwise OLS regression. This form of regression analysis adds variables gradually, till the best level of fit of the model to the data is achieved. This process has two, basic, different iterations. That of

forward selection and that of backward elimination. In our case we have used the forward selection process. This results in several variables excluded from our original model, a phenomenon that can be attributed to the multicollinearity issues that we have shown to be pervasive in our original dataset. PCA will be employed to counter the effects of multicollinearity, as we have already explained in the methodology section.

Although the results appear to be overly good as all variables are statistically significant at the 1% and 5% level, however one will quickly notice that in both samples, there have been eliminated variables. Specifically, for the US Dividend Growth, Big4 Auditors, Capital Expenditure, Risk management practices, Alpha and Beta have been removed, while for the UK the list is even more extensive, with prominent variables such as intangible assets, FCFF, alpha, FCFE, ROE, Beta, among others, not being present. This is something that, as we have explained, we wanted to avoid, as removing variables might lead to a model misspecification. It is however a strong indication that multicollinearity exists, within our original variables.

Various measures have been proposed to resolve the effects of multicollinearity. The first line of defense is the construction of the model itself. By excluding variables that are known for their linear combinations, before they are included in the model. In the same way, another solution more commonly promoted is that of excluding specific variables that exhibit high correlation with others within the model. As already discussed, that measure includes the risk of eliminating a valuable source of information, and subsequently reduces the explanatory power of the model. Other methods, more radical in nature, include the alteration of the data, by the inclusion of alternative forms of the same data or even by including another dataset. However, an approach which is effectively and efficiently applied is Principal Component Analysis, which constitutes the main analytical distillation modelling technique used in this thesis.

5. Principal Components Analysis and Private Company Results

5.1 PCA Results

5.1.1 UK

Both methodologies employed in the previous section (OLS and Stepwise OLS), reveal that our original model is affected by multicollinearity, as indicated for the first one by the high VIF values and by the omission of several variables from the latter. Having seen the effect of multicollinearity on the model, one can easily understand why it is imperative to deal with it rigorously. In order to achieve that, the PCA methodology will be utilized. The first step would be to determine how many components can be used to describe the sample and what "weight" each one of them bears on it. Table 18 reports the Eigenvalues (as defined in section 4.4.2.1.) of the most important Principal Components (PCs):

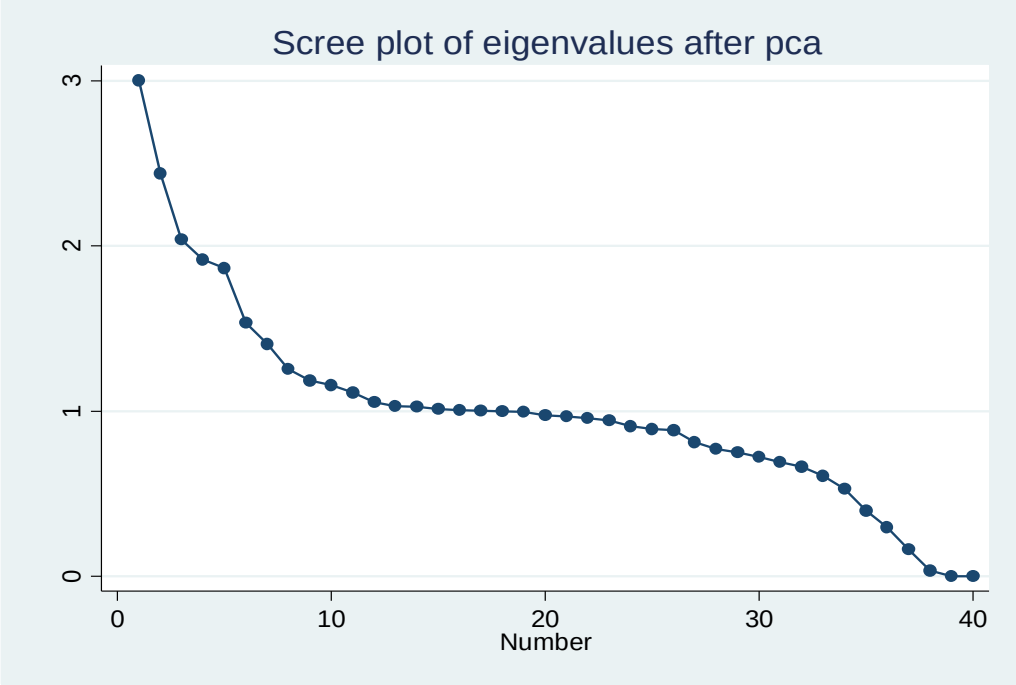
UK Eigenvalues

Component	Eigenvalue	Proportion	Cumulative
Comp1	3.0023	0.075	0.075
Comp2	2.4393	0.061	0.136
Comp3	2.0396	0.051	0.187
Comp4	1.9189	0.048	0.235
Comp5	1.8662	0.046	0.281
Comp6	1.5323	0.038	0.320
Comp7	1.4065	0.035	0.355
Comp8	1.2555	0.031	0.386
Comp9	1.1856	0.029	0.416
Comp10	1.1573	0.028	0.445
Comp11	1.1120	0.027	0.472
Comp12	1.0536	0.026	0.499
Comp13	1.0306	0.025	0.525
Comp14	1.0255	0.025	0.550
Comp15	1.0133	0.025	0.576
Comp16	1.0064	0.025	0.601
Comp17	1.0007	0.025	0.626

Table 18: This table indicates the eigenvalues and the proportion of the variation explained as well as the total variation explained by the components

Overall the 45-variable dataset[57] can be reduced to a new 17- component variable dataset, as it is indicated by the Eigenvalues table. All of these components account for a total of 62% of the variation within the original dataset, with the largest component (Comp1) accounting for 7.51% and the smallest (Comp17) for 2.5%. We chose those components with an eigenvalue of over 1, following Kaiser's rule explained below and as indicated in the relative literature (Callahan et al., 2003; Jolliffe, 1986; Madrid-Guijarro et al., 2009).

Another tool we have for the selection of the appropriate number of components to represent the original dataset, that can be used carefully and in conjunction to the Eigenvalues table, is the Scree-plot (Jolliffe, 1986).



[57] We exclude the macroeconomic variables as they are going to serve as control variables, following the methodology of Callahan, Millar, and Schulman (2003). Control variables are variables that are separate from the variables of interest (the ones we want to focus primarily on) but have been proven throughout the literature that affect our dependent variable. Within the premises of this thesis, the macroeconomic variables act as control, as the overall conditions (at both a national and international level) are similar for all the companies within our samples, and as such the effect on them will be similar. As such they are excluded from the PCA analysis but are included in the fixed-effect regressions for the AIM and NASDAQ companies, as well as the multiple linear regression analysis of the private firms, that are conducted in the following sections.

Figure 9: Scree plot of the eigenvalues produced by the PCA on the UK dataset

When using the Scree-plot, the point where the slope of the graph exhibits a sharp decline or, as more commonly referred to, an "elbow", defines the point from which the components become less relevant. However, a significant problem in interpretation arises when instead of a sharp decline in the slope there is a steady one. That is the reason, why the Scree-plot should be used jointly with the eigenvalues.

Although in this particular case, Figure 9 may not be extremely clear at which component to set the cut-off, if it is examined with the previous table in mind, we will notice that the there is a steep decline up to the 10th component. After that the slope of the line becomes less steep, which indicates that the components at this point exhibit a smaller eigenvalue and thus explains less of the variation in the sample. The reason the components after the 17th are not taken into account despite all of them seemingly being close to the value of 1[58], is because components 18 and 19 (not displayed in this table) have values below that of 1 (0.9988 and 0.9953 respectively).

To give some further insight on the reasoning behind the cutoff we will explain what is commonly referred to in the related literature, as Kaiser's rule, which is a notion based on the paper of Kaiser (1960). The rule suggests that since all the elements in the original set are independent, their representative components exhibit similar characteristics, and specifically variance equal to 1. So, any component with an eigenvalue less than 1 will retain less information from the original set and subsequently needs to be abandoned. Other studies have advocated towards a lower threshold, however the consensus in the literature follows the aforementioned rule.

Having determined the number of components the composition of the components that describe the original dataset are examined. The results for the UK are presented in the Tables 19 and 20. An initial view of the results will reveal that the cut-off point for the variables' coefficients has been set to 0.40. This means that, in order for the variables to be examined more carefully, they need to contribute to above 40% of the

[58] Based on the Kaiser's rule.

variation of the component. The cut-off point for the coefficients, is usually determined on the contextual basis of the research being conducted, depending on the needs of the researcher, with the ranges varying from 0.3 to 0.7 (Joliffe, 1986).

For the first component, in the UK AIM, it can be noted that three variables exhibit the most highly significant coefficients, namely FCFF (0.5759), FCFE (0.5042) and Capital Expenditure (0.5226). This component is dedicated to one of the most common contributory elements used in DCF valuation, namely the free cash flows. The importance of the FCF variables has been highlighted in several studies, for example in Gaspar and Massa (2007) and Kyriazis and Anastassis (2007), who associate them with liquidity and market value respectively. These show to the analysts, what cash is available to firms after they have invested their capital to acquire new assets, and whether this cash is redirected to the firm's equity and debt owners (in the form of interest and debt repayment), or if it is simply available to pay as dividends to common stockholders. It is to be expected that these variables are more important in the case of AIM companies, as these companies are less liquid, more risky investments and thus are expected to provide investors with clear signs on their ability to generate their expected returns.

PCA Results UK								
	Components							
	Component 1	Component 2	Component 3	Component 4	Component 5	Component 6	Component 7	Component 8
Variable								
Financial Crisis		0.4463						
Volatility								
IAS 34 / 2014								
Companies act 2006		0.4944						
IAS Adoption 2005		0.4404						
Effective Tax Rate								
Big 4 Auditors								
Intangible Assets				0.6561				
Tobin's Q								
Risk Management Practices								
Assets								
EBITDA								0.4204
R - Square								
ROE						0.4926		
ROC								
Retention Ratio								
Depreciation				0.6531				
Earnings Yield								
Debt to Equity							-0.4592	
Enterprise Value								
Insiders Stock								
Board Composition								
Compensation to Assets								
No. of Insiders Holding Stock								
Inventory to Sales								
Net operating Margin								
Dividend Growth								
FCFF	0.5769							
FCFE	0.5042							
Capital Expenditure	0.5226							
WACC			0.4826		-0.4828			
Cost of Equity			0.4904		-0.4647			
Cost of Debt								0.419
Z-Score								
MPK (Industry)								
ROA (Industry)								
IK (Industry)								
Alpha								
Beta								
Total Beta								

Table 19: UK PCA Results. Each component is built up by a linear combination of the original variables, with some of them contributing more to the component than others. In this table we report the variables that contributed significantly to each component, as this is indicated by a weight of above 0.4. Also, the contribution to the component of each original variable is marked as positive or negative by the sign in front of each weight.

Following on from this, the second component combination is focused on three key elements. The first one is the financial crisis of 2008 (0.4463), that originally hit the US but through the banking sector spread throughout the world and expressed itself as both a banking and a fiscal crisis, in varying forms. The second and third elements revolve around changes in legislation AIM companies have been faced with. Specifically, the Companies Act of 2006 (0.4944) and the IAS adoption in 2005 (0.4404). Increased

regulatory requirements provide greater transparency for shareholders and are generally viewed as value increasing. For instance, Pasiouras, Tanna, and Zopounidis (2009) find that legislation that increases supervision and control over the banking sector are linked with higher cost and profitability efficiency. Hail (2013), also attests to that notion, as he finds that the significant changes in the regulatory environment in countries with high disclosure requirements, can be linked to higher firm valuations, as the accounting information available become more relevant to outside investors. This idea can act as the link that connects the variables that constitute this component.

The third component focuses primarily on the discount rates that the firm faces, in the form of its cost of capital, with the cost of equity being brought to the spotlight. WACC, with a coefficient of 0.4826, is one of the two major contributors to this component, with the other one being the Cost of Equity (0.4904). The fact that the cost of capital is among the 3 first components, and subsequently among the largest contributors to the variation in the discount rates, is not surprising. It has been established both by academics and professionals alike (see for example studies such as Damodaran 2005, Fernández 2011 and Krüger et al. 2015) that WACC is a good measure of the risk associated with a firm and its ongoing projects. Furthermore, the cost of equity, which is the required rate of return for investing in a firm's stock, is also highly related to the firm's risk level. These two variables also reappear in the fifth component and regardless of the sign of their coefficients[59], this is a clear indication of how important these two variables are as determinants of the P/E ratio and thereby value.

The fourth component is comprised of two variables. Intangible Assets, with a coefficient of 0.6561 and Depreciation, which has a coefficient of 0.6531. Intangible assets are non-physical assets and can be divided into identifiable and unidentifiable. The former are rights, on patents for example, that can be sold, while the latter are embedded within the company's value and their useful life can be inexhaustible, with goodwill

[59] The sign of the coefficients is not important in explaining the overall sample, however they are important within the components, as they denote the effect they have upon each other and the component and as such they need to be explained together with the rest of the variables within the component they are in. For the purposes of this thesis however it is not important that the signs are explained as we are only interested on how the components are created and not on the interaction between the components' contributors.

being the primary example of that. It is also worth noting that goodwill on the balance sheet is usually the result of an acquisition, and an increased goodwill can be an indication of an increased possibility of being an acquisition target (Chauvin and Hirschey, 1994). Depreciation (or amortization in the case of intangible assets) being a non-cash expense affects positively the cash flow of a firm. This indicates that this component is descriptive of how intangible assets affect a firm's cost of capital, together with the free cash flow increase by their amortization.

The sixth component is primarily determined by one variable, ROE, with a coefficient of 0.4926. The profitability of a company is an important aspect, perhaps one of the most heavily focused and subsequently researched even, as it measures the ability of a company to generate sufficient income based on its assets, capital and equity. Return on equity, being one of the factors that analysts heavily rely on to determine a company's profitability, Laitinen (2002) suggests, it is one of the most commonly used measure of shareholders profitability ratio), explains about 4% of the total variation (as it is the primary contributor to the 6th component) in the discount rate sample for the AIM companies. ROE can also be used in comparison to the cost of equity, as a measure of the profits that the company is making and how those relate to the expected returns. Kothari et al. (2006), explain that a company with higher return on equity than cost of equity is associated with higher expected returns.

The seventh component reflects the importance that leverage has on determining the company's risk extent. Debt-to-Equity ratio (-0.4592), as all solvency ratios, examine the company's abilities to meet its long-term obligations, and is one of the most important ratios taken into consideration by analysts. As discussed in the Data section it has been mentioned initially by Bowman (1979), and has been referenced throughout the literature ever since. It is very important, as its fluctuations exhibit how reliant a company is on raising capital through debt. Increased amounts of leverage might indicate financial distress, as a great amount of debt will make it harder for the company to meet its obligations.

The eighth component is a linear combination between EBITDA (0.4204) and the Cost of Debt (0.419). The Cost of Debt represents the market interest rate on new debt that a company might issue, and it might

be affected by a number of company characteristics, some of which might be associated with how the debt itself is structured. In the case of private enterprises determining the cost of debt might rely on estimations based on the firm's existing debt and if available, a rating. EBITDA is a profitability measure, and is an element used commonly in financial analysis (for example in profitability ratios such as the operating profit margin), to examine the ability of a company to generate free cash flow before meeting its obligations to debt holders. EBITDA has often been used as a proxy for cash flow. Liu, Nissim, and Thomas (2002) argue that EBITDA is a cash flow measure that should be linked to the enterprise value, which they define as debt plus equity. As such, it is easy to see the link between debt and EBITDA, which is associated with the total cash generated and the tax-shield on debt.

The ninth component is characterized by the relationship between the effective tax rate (0.4466) and Tobin's Q (0.4621), which has gained ground over the recent years as a measure for company growth or profitability or as an overall measure of how the market values the company (Core, Holthausen, and Larcker, 1999; Ng and Rezaee, 2015). The effective tax rate is defined as the income tax expense over the pretax income. What determines the variation within the effective tax rate, is not only the differences between different states of operation, but also the occurrence of a permanent or a temporary difference in the taxable and pretax income. The difference between the two is that the former is not expected to change in the foreseeable future, while the latter are only affecting the company in the form of either a deferred tax asset or a liability, which is reflected in the balance sheet. These differences will result in tax deductions or taxable income and that is the reason they affect the valuation of companies. As Tobin's Q is essentially the market value of a company over its assets, changes in the latter will be incorporated in the ratio. Thus, it is safe to assume that this component is mainly associated with the market value of the firm and how it is affected by any gains that might be realized through tax-related occurrences.

The tenth component is one of the most interesting, as it features the relationship between two key variables related to the cost of capital, but with the further addition of an extra variable that was relatively recently adopted to account for the additional risk faced by investing in private enterprises (or in this case public

companies that face similar restrictions as private companies). The first one is R-Square (-0.4597), which simply indicates the OLS coefficient of determination in the longitudinal relationship between the company's stock and the market index. The second one is the CAPM Beta (0.4647), which indicates the volatility of the stock relative to the volatility of the market the firm is operating in and is most commonly used as an indicator of systematic risk by analysts. The variable that is the most interesting however in this component is the one with the highest coefficient namely Total Beta (0.6798). When considering this variable, it is important to remember that AIM market companies were selected as they best approximate private enterprises, mainly because they face similar illiquidity restrictions, non-diversified investors, as well as having similar close insider ownership structure.

PCA Results UK (Continued)									
	Components								
	Component 9	Component 10	Component 11	Component 12	Component 13	Component 14	Component 15	Component 16	Component 17
Variable									
Financial Crisis									
Volatility									
IAS 34 / 2014									
Companies act 2006									
IAS Adoption 2005									
Effective Tax Rate	0.4466								
Big 4 Auditors							0.6337		
Intangible Assets									
Tobin's Q	0.4621								
Risk Management Practices									
Assets				0.5472					
EBITDA									
R - Square		-0.4597							
ROE									
ROC									
Retention Ratio									
Depreciation									
Earnings Yield								0.6374	
Debt to Equity									
Enterprise Value									
Insiders Stock					0.4865			0.5644	
Board Composition									
Compensation to Assets									0.6646
No. of Insiders Holding Stock			-0.6789						
Inventory to Sales			0.6595						
Net operating Margin									
Dividend Growth									
FCFF									
FCFE									
Capital Expenditure									
WACC									
Cost of Equity									
Cost of Debt									
Z-Score						0.5193			0.7152
MPK (Industry)									
ROA (Industry)				0.4479		0.4631			
IK (Industry)									
Alpha					0.5185		0.4492		
Beta		0.4647							
Total Beta		0.6798							

Table 20: PCA Results UK

CAPM Beta is a measure of the relative risk added to a portfolio by the inclusion of a new stock. Total Beta was proposed by Damodaran (1999), who argued that since investors in private enterprises are not sufficiently diversified, Beta is not an appropriate measure of the risk these investors face. Consequently, a measure of the private company's total risk (systematic and unsystematic) relative to the overall market's risk is needed and that is Total Beta. As we have already seen in the preface of this section, the AIM companies were selected to best approximate private companies. As such, it was anticipated that Total Beta would be among the important variables that define the discount rate for the companies in the sample.

Moreover, this is, the first study that provides empirical evidence to support the importance of Total Beta, as this measure is accepted by business valuation practitioners (Butler, Schurman, and Malec, 2011) but has not found significant support outside that sphere of influence.

The eleventh component is comprised of two variables, the Number of Insiders Holding Stock (-0.6789) in the company and the inventory-to-sales ratio (0.6595). The number of people from within the company that own the stock of the company can be used in the context of both corporate governance and firm performance. Specifically, the structure of the ownership of the company by insiders is very significant, as it indicates implications in transferring the control of the company to other investors. This is a particularly significant factor in the valuation of private businesses, or in this case of companies with similar characteristics to them. Moreover, it is an indication that the interests of investors and management are aligned, in the sense that management will try to maximize the value from their shares by achieving a good performance for the firm. The inventory-to-sales ratio is a performance measure that demonstrates how quickly the company creates revenues by selling their inventory. The linear relationship between these two variables, however, is unclear, although one would argue that both of them are indicators of company performance, and internal efficiency in operations, however they are used in a different context.

The twelfth component illustrates the relationship between a company's Assets (0.5472) and the Return-on-Assets (0.4479), with the additional adjustment proposed by Cooper and Priestley (2016), (they calculated that ratio by subtracting the average of the industry from ROA) to account for the average variation in the industry. The assets of a firm are very important in the valuation process. This balance sheet item is not only used in several ratios (one of which is ROA), but it also allows appraisers to roughly estimate the value of a company (Asset based valuation) by simply subtracting the liabilities from the assets. Moreover, this item has been used in debt-related research (see for example the papers of Clatworthy, Mellett, and Peel, 2002; Güntay and Hackbarth, 2010), as an indicator of how well a company may serve its debt. Return-on-Assets is one of the most commonly employed ratios, as a measure of the profitability of a company. It links the company's total assets to the net income, however not considering the tax rates

on the interest paid, poses the danger of excluding significant information for the returns of equity and debt holders. The common element that is featured in this component is the importance of the assets in valuation.

The thirteenth component is more heavily influenced by the percentage of stock that insiders hold (0.4865) and the company's Alpha (0.5185). The amount of stock that employees of the company hold is used as an indication of management performance, which is of particular interest in a private company setting. As investors require managers to place shareholder's wealth maximization above anything else, they are able to actively control management performance, and if necessary, proceed to change the manager. In private enterprises though this is not the case, as more often than not, management and ownership are intertwined. Consequently, whoever owns a portion of the stock that is linked to controlling interests over the company can dictate who will be managing the firm. Alpha (or Jensen's Alpha, as it is most commonly referred to in the literature), is also a measure of management performance (Jones and Shanken, 2005), as it indicates the abnormal returns associated with an asset. One could suggest that this component, with its 2% explanatory power, is primarily targeting concerns on management performance.

Furthermore, the fourteenth component is derived by the combination of Altman's Z-Score (0.5193) and a variable we have seen in the twelfth component, namely ROA (0.4631). The Z-Score, which was originally proposed by Altman (1968) examines the probability of a company going bankrupt. It focuses primarily on the solvency of the company, by examining different aspects of the company's profitability, leverage, liquidity and activity. Five of these ratios, used in the calculation formula, are premised upon or relative to the company's assets. This is the reason that ROA is resurfacing as a significant variable within this component. Once again, the importance of a company's assets is highlighted, as a heavily asset backed company, especially in the case of AIM or private ones, might be a signal of higher liquidity potential to investors and longer-term security.

The fifteenth component focuses primarily on the auditors of the company (0.6337) and secondarily on the firm's alpha (0.4492). As indicated in studies such as Henock (2005) or De Franco et al. (2011) the use of one of the 4 big auditing companies serves as an indication to analysts, that all the financial statements will

accurately reflect the financial situation of a company, and will also serve as a deterrent against manipulation from management. Additionally, Gerakos et al. (2013) report significant and positive coefficients for the returns of AIM listed companies that use one of the big auditing companies. Subsequently, it is to be expected that one of the components would be focused on the relationship of management performance and the auditing of the firms.

Additionally, the sixteenth component is majorly affected by the earnings yield (0.6374) and the amount of stock the insiders of the company hold (0.5644). The earnings yield is calculated by dividing the earnings per share with the market price of the company's stock. It allows analysts to determine one perspective of whether a stock is potentially overvalued or undervalued. Furthermore, the reappearance of the stock held by insiders of the company signals the significance of this variable for the AIM companies. In general, this component can be viewed as an image of the connection between firm performance measures.

Finally, the seventeenth component indicates a relationship between the Compensation-over- Assets variable (0.6646) and Altman's Z-Score (0.7152). Overall, the compensation can be an indication of how management's incentives are aligned with those of the investors. Furthermore, in private or other illiquid firms (AIM), an analyst when applying certain valuation methods such as a discretionary earnings approach, is required to adjust operating income for the compensation expenses of the manager of the firm, who in many cases will also be the owner. These adjustments are important, because they reflect what Damodaran (2002), refers to as the substitution cost of these managers, because a valuation, as he explains, must contain the appropriate compensation expense for them to be able to be replaced. Additionally, as compensation might include items, other than salary, further alterations to operating income must be made (Damodaran explains how in private or small businesses a mixing of personal and corporate expenses exists, which can cause the actual operating income of the company to be miscalculated, and subsequently lead to a higher or lower value appraisal). Therefore, we might conclude that this component reflects how compensation might lead to higher discount rates.

By applying PCA to the original dataset it has significantly reduced the number of key variables from the original number of 45 to a total of 17 new variables (which are linear combinations of the original variables), with which we will create an index that will be used in the following section of the panel regression analysis. One could view the eigenvalues associated with each one component as the weight that can be assigned to each one of them. Furthermore, each of those components can be described by a few variables, which usually have some value-based connection to one another. Although some of them are seemingly more abstruse and complex, the underlying idea on how they are formed is the main focus of this thesis, as it facilitates the association of specific variables with their importance in the formation of the discount rate.

5.1.2 USA

Following the same methodology as that applied with the UK sample, the PCA method is employed on the US data and the eigenvalues are presented in Table 21:

US Eigenvalues

Component	Eigenvalue	Proportion	Cumulative
Comp1	3.30682	0.0827	0.0827
Comp2	3.19870	0.0800	0.1626
Comp3	2.93518	0.0734	0.2360
Comp4	1.93473	0.0484	0.2844
Comp5	1.84220	0.0461	0.3304
Comp6	1.26174	0.0315	0.3620
Comp7	1.20918	0.0302	0.3922
Comp8	1.19441	0.0299	0.4221
Comp9	1.14602	0.0287	0.4507
Comp10	1.09303	0.0273	0.4781
Comp11	1.07545	0.0269	0.5049
Comp12	1.05558	0.0264	0.5313
Comp13	1.04878	0.0262	0.5575
Comp14	1.01376	0.0253	0.5829
Comp15	1.00461	0.0251	0.6080
Comp16	1.00141	0.0250	0.6330

Table 21: This table reports the eigenvalues, the portion of the total variation explained as well as the total variation of the components

The original dataset can be reduced in size significantly, from, again, a 45-variable one, down to a total of 16 variables, which account for approximately 63.3% of the total variability of the original one. The largest component explains 8.27% percent of the variability overall, with the second one following closely to 8%. There is a gradual decline for the eigenvalue of the third component that becomes sharp for the fourth and then gradual again from this point onwards. The sixteenth component, being the smallest one, explains 2.5% of the variation of the original sample. One thing that can be noted by viewing the last 8 components is that they are almost equal to the percentage of the variability they explain (most of them being in the 2.5 to 2.8%). All of the above is depicted graphically in the following Scree Plot (Figure 10):

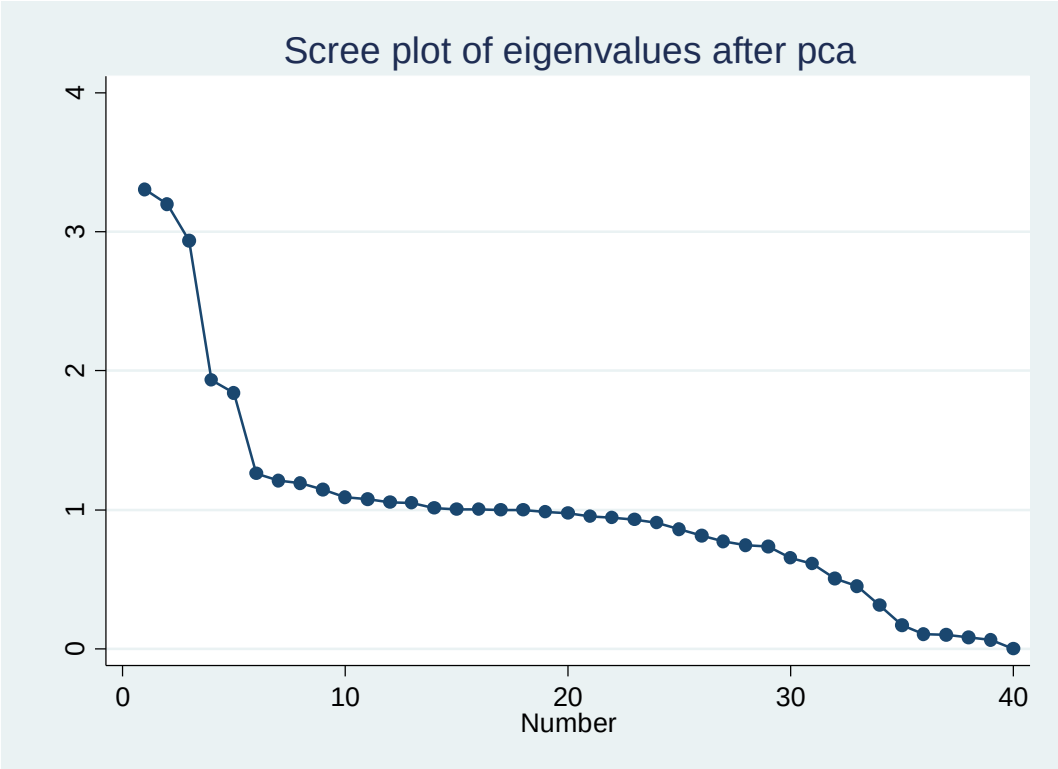


Figure 10: Scree plot of the eigenvalues produced by the PCA on the US dataset

In this test context the rapid decline in the eigenvalues and the subsequent creation of the "elbow" is more clearly displayed. From the first to the second component the decline is smooth, however it becomes more

rapid from the third to the fourth, after which it momentarily smooths out again, until it finally becomes gradual, as the eigenvalues get closer to Kaiser's cut-off Eigen value of 1 at the 16th component.

As the number of components to be retained are identified the next step is to proceed to view which variables contribute the most to the creation of the linear combination of the components. The minimum threshold of 0.4 for the coefficient to be reported as an important contributor to the creation of a component is again applied here. The results can be found in Tables 22 and 23, which are reported below:

PCA Results - USA								
	Components							
	Component 1	**Component 2**	**Component 3**	**Component 4**	**Component 5**	**Component 6**	**Component 7**	**Component 8**
Variable								
Financial Crisis		0.4216						
Volatility								
Jobs Act								
FAS 123R								
Entry Amendmends								
Effective Tax Rate								
Big 4 Auditors								
Intangible Assets					0.4141			
Tobin's Q								
Risk Management Practices								
Assets					0.5179			
EBITDA	0.4234							
R - Square							0.4216	
ROE								
ROC								
Retention Ratio								
Depreciation	0.4173							
Earnings Yield								
Debt to Equity								
Enterprise Value								
Insiders Stock								
Board Composition								
Compensation to Assets			0.5809					
No. of Insiders Holding Stock								
Inventory to Sales						0.669		
Net operating Margin						-0.6703		
Dividend Growth								
FCFF								
FCFE								
Capital Expenditure	-0.4235							
WACC				0.6561				
Cost of Equity				0.6504				
Cost of Debt								
Z-Score			0.5708					
MPK (Industry)								
ROA (Industry)			0.58					
IK (Industry)								
Alpha								
Beta								
Total Beta								

Table 22: US PCA Results Each component is built up by a linear combination of the original variables, with some of them contributing more to the component than others. In this table we report the variables that contributed significantly to each

component, as this is indicated by a weight of above 0.4. Also, the contribution to the component of each original variable is marked as positive or negative by the sign in front of each weight.

The first component in this test is comprised of EBITDA (0.4234), Depreciation (0.4173) and Capital Expenditure (-0.4235). The relationship between EBITDA and Depreciation can easily be understood, as EBITDA, commonly used measure of profitability, represents operating profit with the addition of Depreciation and Amortization. Both of those variables, however, exhibit an inverse relationship to Capital Expenditure as it is indicated by its negative sign. The relationship is inversed probably because capital expenditure refers to the amount of resources a company spends to acquire new physical assets, which reduce its operating income and furthermore are subject to depreciation. Moreover, EBITDA is a proxy for cash generated, depreciation is also a measure of a non-cash cost and Capital Expenditure is part of the calculation to arrive at FCF thus this combination is related to Free Cash Flow. Subsequently, one could posit that the first component illustrates how a company's generated cash flow is used on equipment, as an indicator of future growth.

The second component is dominated by one variable, namely Financial Crisis (0.426). It can easily be understood why the financial crisis of 2008, holds such an important role in explaining the variability of the discount rate components. To expand on this notion, this exogenous shock, initially reduced significantly the access that companies had to funding sources, as firms were having difficulties in raising capital through debt, as banks and other creditors were attempting to reduce their risk profile and stabilize equity. After Barack Obama assumed the presidency however, and through a series of favorable, to smaller and medium enterprises, legislative acts (JOBS Act), provided the markets with incentives to increase the financing liquidity of these firms. This resulted in an increased volume of new companies entering the stock markets in search of capital (Dambra, Field, and Gustafson, 2015).

The third component is a linear combination of three variables, namely the Compensation-to-Assets ratio (0.5809), the Z-Score (0.5708) and the Return-on-Assets (0.58). Besides the obvious link that these three variables share, namely how their various elements behave around the assets of a company, it can be seen

that there is another link between them. The first one is an indication on whether the management's interests are aligned with those of the investors, as this becomes evident by both the type of compensation (whether it is incentive-based or fixed) but also the magnitude of the compensation as a proportion of the company's assets. Moreover, an investor will understand how capable a manager is to create earnings based on the company's assets. If those two indicators are not providing investors with good signaling then it is possible that the company will face solvency issues, which will be depicted in its Z-Score.

The fourth component highlights a well-established relationship between a company's Weighted Average Cost of Capital (0.6561) and its Cost of Equity (0.6504). Specifically, what is pointed out here is that for NASDAQ company, the cost of equity is a more important factor than the cost of debt in the calculation of the overall cost of capital that this company faces. Furthermore, the fifth component clearly illustrates the importance of the assets, with the intangible being even more important (which as mentioned previously might be an indication of the company becoming an acquisition target and thusly affect the risk profile of said company), in the determination of the discount rate. The two major variation contributors to this component are the Intangible Assets (0.4141) and the Assets (0.5179).

PCA Results - USA (Continued)								
	Components							
	Component 9	Component 10	Component 11	Component 12	Component 13	Component 14	Component 15	Component 16
Variable								
Financial Crisis								
Volatility								
Jobs Act								
FAS 123R								
Entry Amendmends								
Effective Tax Rate							0.6735	
Big 4 Auditors								
Intangible Assets								
Tobin's Q	0.45							
Risk Management Practices		0.5748						
Assets								
EBITDA								
R - Square								
ROE								
ROC					0.4814			
Retention Ratio								
Depreciation								
Earnings Yield								
Debt to Equity				0.5825				
Enterprise Value								
Insiders Stock						0.8149		
Board Composition		0.448						
Compensation to Assets								
No. of Insiders Holding Stock								
Inventory to Sales								
Net operating Margin								
Dividend Growth								
FCFF			0.4853					
FCFE			0.5245					
Capital Expenditure								
WACC								
Cost of Equity								
Cost of Debt								
Z-Score								
MPK (Industry)								0.4553
ROA (Industry)								
IK (Industry)				0.5851				
Alpha								-0.7078
Beta							0.5378	0.4995
Total Beta								

Table 23: US PCA Results continued

The variables that dominate the sixth component, are the Inventory-to-Sales ratio (0.669) and the Net Operating Margin (-0.6703). A first observation that can be made here is the opposite effect these two variables have on the component denoted by the opposite signs of the components' dominant constituents. As we have already seen Inventory-to-Sales is an activity ratio denoting efficiency, indicating to analysts the rate at which a company replaces its inventory over a period of time. Net Operating Margin reveals how operating profit, or a company's earnings, fare over its sales. Both those variables have the sales (or the

revenue stream of the company) as their link, which is a clear indication on how important revenues are for the analysts.

The seventh component is clearly defined by only one variable, CAPM's R-Square, while the eighth has no prevalent variable (all variables' coefficients are below the set threshold), which is an indication of equal distribution of the variability by the constituents of the component, at lower levels. Furthermore, the predominant variable in the ninth component is the market value of the firm as represented by Tobin's Q (0.45). This variable is extremely important, as it is featured in a series of valuation papers (Allayannis and Weston, 2001; Gozzi, Levine, and Schmukler, 2008; Roll et al., 2009).

The tenth component can be viewed as a corporate governance composite as it features the Risk Management Practices (0.5748) and the Board Composition (0.448). Risk Management Practices are especially useful as they act as a hedge against adverse events, which might influence the company at various levels, ranging from operations to the management itself. Specifically, companies tend to link their top executives compensation to their performance, mainly through the adoption of equity options instead of other forms of fixed compensation, such as salary or bonuses (Bergstresser and Philippon, 2006; Williams and Rao, 2006). This control over the performance of the top management team can also be expressed through the Board Composition, namely how many internal and external members participate in the company's board of directors and how many of these members participate in the various committees that oversee the smooth operation of the company. This component highlights the importance of corporate governance practices to a company's cost of capital.

The eleventh component illustrates the importance of the cash flow generated by the firm for the debt holders, as well as the common stockholders and it is expressed through FCFF (0.4853) and FCFE (0.5245). As we have already seen in the previous section, where the UK AIM was evaluated, both the variables that represent the Free Cash Flow are a key measure that is used by appraisers. Additionally, two variables constitute the twelfth component, namely the Debt-to-Equity Ratio (0.5825) and the Investment-to-Capital ratio (0.5851), adjusted for the industry average. The former ratio is indicative of how leveraged a firm is

relative to its equity. A highly leveraged firm is indicative of a higher probability of potential default. However, if the leverage is used to fuel capital investment, which is what the latter ratio refers to, it might be an indication of potential growth for the firm. In that sense this component links leverage and growth for a company.

The variable featured in the thirteenth component is the Return-on-Capital ratio (0.4814). This ratio is commonly used by appraisers to assess the profitability of a company, and more specifically, how the short and long-term debt, together with preferred and common equity, are serviced by the earnings of a company (or in simpler terms the margin of safety between ROC and cost of capital and can be indicative of potential excess returns in relation to the cost of capital). The fourteenth component is mostly affiliated to a variable that we have seen in several occasions already, namely the percentage of stock that insiders hold for the firm they work for. The reappearance of this variable is a clear indicator of the importance of the controlling interests within the firm, and how those are translated to the control over its management.

The fifteenth component features another very important aspect of valuation, reflected in the discount rate. The Beta of a stock (0.5378) is directly linked to the effective tax rate (0.6735). The unlevered Beta of a firm is linked to the Beta of other comparable firms over the corporate tax rate adjusted debt to equity ratio. The effective tax rate has been considered in the previous section; it can be seen here that it can also be used in the place of corporate tax rate to calculate the unlevered Beta. Thus, this component states the importance of the effective tax rate and how it can be used by analysts to calculate the systematic risk associated with an investment in an enterprise.

Lastly, the sixteenth component is an amalgamation of three different variables. It is mostly influenced by Jensen's Alpha (-0.7078), the company's relative volatility to the market as this is explained by the Beta (0.4995) and finally by the Marginal Profit-to-Capital ratio (0.4553), adjusted for industry average. The MPK ratio, is a profitability ratio that links long-term financing to how well a company utilizes its assets. This not only serves as a signal on the performance of the firm but can also be viewed as a measure for the survival of the firm in the long run, as firms who are able to utilize their revenues to better service their

long-term debt, have a lower probability to default. One can assume that this component is highlighting how the company's risk is connected to the way that managers best utilize the assets they have at their disposal to yield the best possible performance.

5.2 Comparison between UK and USA

The countries chosen for this study share several common characteristics. Both are developed and mature, with macroeconomic indicators, which act as benchmark for other economies around the world. Similarly, their respective markets are also developed, with NASDAQ featuring companies from every element of the size spectrum, while AIM although smaller in size, is also one of the most successful markets in the world (Gerakos et al., 2013). Both act as a significant liquidity stream to businesses, with AIM consisting mostly of small and medium sized enterprises, with high growth potential, while NASDAQ companies being more established and at a later stage in their life cycle. It is to be expected that both these markets have several common factors that affect their valuation, however they both exhibit significant differences, which can be attributed to the different characteristics of the firms that are listed on them.

Before proceeding to contrast the structure of the components, some qualitative observations can be drawn. Firstly, the PCA for both countries, has revealed structural differences on the discount rate datasets. The number of key components needed to explain the discount rate variability differs, with the UK needing 17 components, while the US needs 16 components. This is signaling that regardless of the common characteristics, these markets might share, investors look at different variables to determine their expected returns.

Furthermore, the proportion of the variation explained by the first three components in the US is significantly higher than the one explained by the first three in the UK, as consequently so does the cumulative variation. However, the UK components' variation seems to be more evenly distributed amongst the components, something that is also marked notably by the smoother curve in their respective scree plots.

This is an indication of a less discrete connection between the variables that form the components. Additionally, the sharp decline in the eigenvalues of the US data, results in a smaller number of components that explain a larger part of the original dataset, with more than half of the components in this case explaining less than 27% of the total variability, while this is reversed for the UK. All of the above can be seen in Table (24) that follows.

Component's Constituents							
	UK				US		
Component	Variables				Variables		
1	FCFF	FCFE	Cap. Exp.		EBITDA	Depreciation	Cap. Exp.
2	Financial Crisis	Companies Act 2006	IAS Adoption 2005		Financial Crisis	-	-
3	WACC	Cost of Equity	-		Compensation to Assets	Cost of Debt	ROA (Industry)
4	Intangible Assets	Depreciation	-		WACC	Cost of Equity	-
5	WACC	Cost of Equity	-		Intangible Assets	Assets	-
6	ROE	-	-		Inventory to Sales	Net Oper. Margin	-
7	Debt to Equity	-	-		R-Square	-	-
8	EBITDA	Cost of Debt	-		-	-	-
9	Effective Tax Rate	Tobin's Q	-		Tobin's Q	-	-
10	R-Square	Beta	Total Beta		Risk Management Practices	Board Composition	-
11	No. Insiders Holding Stock	Inventory to Sales	-		FCFF	FCFE	-
12	Assets	ROA (Industry)	-		Debt to Equity	IK (Industry)	-
13	Insider's Stock Percentage	Alpha	-		ROC	-	-
14	Z-Score	ROA (Industry)	-		Insider's Stock Percentage	-	-
15	Big 4 Auditors	Alpha	-		Effective Tax Rate	Beta	-
16	Earnings Yield	Insider's Stock Percentage	-		MPK (Industry)	Alpha	Beta
17	Compensation to Assets	Z-Score	-				

Table 24: This Table presents the constituents of each component for both the US and the UK.

Moving forward to the individual components, what explains the largest part of the UK variation in the discount rates, as reflected in the first component, are the Free Cash Flows, while the same position is being held by the EBITDA in the US (which is considered as a proxy for cash flow). Moreover, the external shocks, as those are approximated by the Financial Crisis dummy variable, are solely incorporated in the second component, while the UK second component, adopts a more generic view and besides the Financial Crisis includes legislation changes. This might be attributed to the fact that AIM companies, are smaller in size and more illiquid, and are subsequently more affected by the changes in what they might be required to disclose.

The US third largest component is related to the assets of the company, as those are examined through the prism of the total assets and the return on total assets, while simultaneously these assets are markers linked to the measure of probability that a company might be at higher risk of bankruptcy. This is not the case, however for the UK companies, that their third most significant component is dedicated to how much they pay for their capital, with an extra focus on the capital raised through the equity. Again, this seems to be linked to the fact that AIM companies are less liquid, and subsequently an appraiser will focus more on how these companies finance themselves. For the NASDAQ companies this trend appears in the next component the fourth, while for the same component AIM companies are mostly preoccupied with their Intangible Assets, namely patents, trademarks or even acquired goodwill.

The Assets, both tangible and intangible, are featured in the fifth component for the US dataset. Assets are important for the investors, as they reduce the probability of them losing their initial investment, even if the company does go bankrupt. In the same component slot, the UK sample of the variables affecting the discount rates, once again highlights the importance of the firm's cost of capital, while the sixth one focuses on the profitability of the company by giving prominence to ROE. NASDAQ companies on the other hand focus not only on the profitability but also on their activity, as the Inventory-to-Sales ratio and the Net Operating Margin are the determinants of this component. This differences in the last two components can probably be attributed to the markets that these companies operate in, and what kind of companies those

are. One can expect to see a young start-up company without strong asset backing in the AIM, however the NASDAQ might include more mature companies, with backing from funds or other types of investors.

The seventh component features only one variable for both markets, however AIM's 3% variability is explained by a solvency ratio, while a similar percentage in the NASDAQ is attributed to the systematic risk associated to the company's stock. Again, this difference and the primary focus on solvency for AIM companies can be attributed to the illiquidity these companies face. This characteristic is further illustrated in the eighth component for the AIM sample, which focuses on the Cost of Debt and how the companies in that market allocate their earnings.

The discount rate, in the AIM, is affected, as is shown by the ninth component by the effective tax rate, and all the items derived by it, and the market valuation of the company. This component partially coincides with the equivalent in the NASDAQ, as the discount rate sample is mostly affected by Tobin's Q, as well. The tenth component, for the NASDAQ, shows the interest of investors for the governance of the companies, while the same component for the AIM reveals, how close the AIM and private companies are, by highlighting the risk associated with investing to these firms, as this is shown by the Beta, the R-Square and the Total Beta. This difference between the markets that form the samples, is very important, as the Total Beta does not appear to bear any significance for the NASDAQ companies. By design, Total Beta, should be used as a measure of risk for private enterprises (Damodaran, 2012). This again reflects the difference of AIM to NASDAQ companies. With AIM companies being more similar to private companies being smaller and less liquid in stock trading and with ownership being more closely held than NASDAQ companies, which have, because of their market's history, become mature and stable companies with dispersed and more atomized ownership and full liquidity on stock trading.

The Free Cash Flows only make an appearance at the eleventh component for the NASDAQ companies (and explain roughly 2% of the total variation), while as seen earlier this is not the case for the discount rates in the AIM companies (the FCF variables appear in the first component and explain the largest part of the variability of the discount rates). For this component the AIM focuses on a measure of management's

performance and control over the firm, while simultaneously looking at the activity of the companies. For the twelfth component, AIM focuses on the Assets of the company, much later than the fifth component of the NASDAQ. As we have explained before, this might be attributed to the kind of the companies that are listed in the AIM market, which are younger, smaller firm, which are most probably earlier in their life cycle (growth companies). NASDAQ on the other hand consider the debt and how the capital of the companies' is employed in order to service that debt.

The thirteenth component is also totally different for both markets. For the AIM the focus falls primarily on the control of the company and thereafter to management, while for the NASDAQ it persists on how well the income of the companies is serving their debt. For the fourteenth component, most of the weight in the AIM sample falls on the probability of default of the companies, as AIM companies operate under the constant supervision of their Nominated Advisers (NOMADS) (Gerakos et al., 2013), which act as safeguards for the companies' proper operation. The NASDAQ sample once again illustrates the importance of the control over the management, by heavily aligning this component with the stock held by insiders of the firm.

The final two components for the NASDAQ, illustrate the importance of financial leverage over the company's risk, and the concerns investors might have on how management acts to best serve their interests, by taking the right actions as to properly utilize the assets at the company's disposal. This is made clear by the high coefficients attributed not only to Beta and Alpha, but also to the effective tax-rate (leveraged Beta) and the Marginal Profit-to-Capital ratio. For the AIM companies however, there is a similarity in the sense that investors evaluate positively the performance of the management but also seem to consider it through the prism of other measures, such as for example the compensation, and how that aligns their interests together with the management's.

5.3 Regression Analysis with the Components as regressors

In this section, following the methodology proposed by Boone et al. (2007), Callahan et al. (2003), Madrid-Guijarro et al. (2009), among others, we will utilize the component index created using PCA, in conjunction with a set of macroeconomic variables, to find further evidence of the effect these variables have on the discount rate, primarily on the public companies' and in the next section on the private companies' datasets. Before, that however, it is important to set the parameters on which this analysis will be done. To address any concerns for multicollinearity we have repeated the VIF procedure described in the previous section and excluded long-term interest rates for the UK and short-term interest rates for the US, as those two variables displayed a problematic VIF value.

The first significant element is the panel nature of the data. Panel data provide researchers with a series of advantages over their cross-sectional or time-series counterparts. Gujarati (2015), explains that panel data focus on different behavioral patterns that individuals might develop over a period, as these patterns may not be revealed if the variables are examined under a pure cross-sectional or time-series aspect. Moreover, the unique synthesis of cross-sectional and time series data allows researchers to study the variability of the determinants, with higher degrees of freedom and more efficiency. He also argues that this type of data, mainly due to the characteristics mentioned above, are more suitable for the study of "phenomena such as economies of scale or technological changes".

Our panel dataset is unbalanced, as the number of companies (N) in the sample are not the same as the number of time of observations (T). It is also a short panel, with the number of companies, being greater than the number of time periods. The usual methodologies employed with this type of data, are pooled, fixed-effect and random effect regressions, with each been focused on different underlying ideas and assumptions.

For the purpose of the thesis we will be using fixed effect (FE), as those present us with a series of advantages. Firstly, FE allow researchers to focus on variables that are not static over time. Another characteristic we want to "exploit" is the fact that FE regressions allows us to isolate specific traits from

the variables, or to put it in more straightforward terms, we want to use fixed effects because we believe that there is an attribute that might cause correlation between the observation's error term and the independent variables, thusly refuting the no-correlation condition of the least-squares (viz. : Cov($x_{it}u_{it} = 0$) for the least-squares assumptions to hold (Wooldrige, 2002). Simultaneously, these non-varying attributes are unique to each observation and are not correlated with this observation's other characteristics, as we treat each observation as unique. To examine whether this assumption is true we will conduct a Hausman test.

Another concern we addressed, following the same methodology as Ng and Rezaee (2015), was that of endogeneity. As in the paper mentioned previously, we use a lag regression design, where the control and independent variables are lagged by one period, and as such the issue of endogeneity can be properly addressed. This is the methodology proposed by most of the major econometrics manuals (see Gujarati, 2015; Gujarati and Porter, 2009; Wooldrige, 2002) as well as in several studies with methodologies akin to this research, in the sense of the data type used (such as panel studies by Korteweg, 2010) and Evans and Schwartz, 2013).

The main assumption we make in this thesis, in order to choose the FE, is that there are characteristics that are common throughout the industries the companies operate in, and as such we want to isolate them. This intuition is further strengthened by the studies of Boone et al. (2007) and Ng and Rezaee (2015), who also use fixed effects regressions at the industry and year levels, as a means to discard the time-series correlation present in their data. Besides prior research, and as we wanted to be certain of whether the method of choice is the most appropriate one, we also conducted a Hausman Test, which indicates whether fixed or random effects should be chosen (Wooldrige, 2002), with the results indicating the use of FE regressions (the critical value is highly significant as indicated at the end of Table 25).

Fixed-Effects Regression results			
	UK (Panel A)		*US (Panel B)*
Dependent Variable:	*P/E*		*P/E*
Control Variables			
Consumer Confidence Index	*0.00618*		***4.391****
	(-0.38)		*(4.23)*
GDP Growth	*-1.76*		***5.979****
	(-1.82)		*(6.53)*
Short-term Interest Rates	***-0.0738****		-
	(-3.23)		
Long-term Interest Rates	-		***-9.873****
			(-4.25)
Yield Spread	***-0.0494****		***-9.161****
	(-2.22)		*(-4.12)*
ICRG rating political risk	*0.267*		***-39.67****
	(-1.46)		*(-4.08)*
Unemployment rate	*4.78*		***-15.54****
	(-1.19)		*(-4.91)*
Inflation	*-0.0336*		***-6.234****
	(-0.89)		*(-5.89)*
Output Gap	***-0.0420****		***-11.61****
	(-2.22)		*(-3.46)*
Component Index			
Comp1	***0.0349****	Comp1	***0.434****
	(-7.70)		*(-6.53)*
Comp2	***-0.0201****	Comp2	***-0.624****
	(-2.85)		*(-5.50)*
Comp3	***-0.0498****	Comp3	*-0.0131*
	(-2.51)		*(-0.17)*
Comp4	*-0.0112*	Comp4	***-0.313****
	(-1.07)		*(-20.98)*
Comp5	*-0.0171*	Comp5	***0.683****
	(-1.90)		*(-33.8)*
Comp6	***0.0539****	Comp6	***2.230****
	(-6.68)		*(-24.70)*
Comp7	*0.0067*	Comp7	***-0.252****
	(-0.81)		*(-5.21)*
Comp8	*0.0155*	Comp8	***-0.707****
	(-1.85)		*(-31.41)*
Comp9	***0.015****	Comp9	***0.338****
	(-2.35)		*(-13.66)*
Comp10	***-0.0126****	Comp10	*0.00907*
	(-6.65)		*(-0.14)*

Comp11	***0.0954******	Comp11	***0.287******
	(-9.28)		*(-15.43)*
Comp12	***0.265******	Comp12	*-0.0168*
	(-22.17)		*(-0.66)*
Comp13	***0.0485******	Comp13	***-0.220******
	(-5.87)		*(-8.56)*
Comp14	***0.0722******	Comp14	*0.0217*
	(-6.52)		*(-1.19)*
Comp15	***0.131******	Comp15	***-0.138******
	(-15.05)		*(-5.26)*
Comp16	***0.0732******	Comp16	***0.105******
	(-9.1)		*(-3.77)*
Comp17	***0.0138****		
	(-2.24)		
Constant	*-0.7*	Constant	***-305.3****
	(-0.35)		*(-2.50)*
N	**55971**	***N***	**89701**
t statistics in parentheses *** for p <0.01, ** for p <0.05 and * for p <0.10			
R^2	**0.19**		**0.37**
Hausman Test for Fixed vs Random			
Prob>chi2 =	0.0000		0.0000

Table 25: This table reports the results of the fixed effects regressions on both UK and US data. We also conducted a Hausman test to further solidify the original hypothesis that we needed to use fixed effects as to isolate specific characteristics from the variables. The Hausman test results are significantly close to zero attesting to the original choice (the null hypothesis is that random effects should be used, and the small p-value would suggest that this hypothesis should be rejected in favor of the alternative which is to use fixed effects).

We begin the analysis of the results from the fixed effects regressions reported on Table 25. Panel A reports the results that the set of control variables and the component index from the UK (AIM) sample have on the valuation measure (the natural logarithm of the P/E ratio). The regression outcome suggests that only three control variables affect the valuation of the AIM companies significantly. There is a negative relationship between higher output gap (which as Cooper and Priestley, 2009 explain indicates the difference between what a country produces and what it can produce), short – term interest rates (which is a measure of liquidity provided towards companies, Kiani et al., 2012) and yield spreads (which can signify the overall country risk, Hyde and Sherif, 2010) and the P/E ratio. These results exemplify what investors regard as important for the environment growth companies operate into.

From the component index, the components that affect negatively the valuation measure in the UK are associated with the financial crisis (Comp. 2), and the reasoning behind it is intuitively clear, with increased WACC and Cost of Equity (Comp. 3), a finding that is consistent with Fernández (2011), Total Beta (Comp. 10), which is a measure of the correlation of the company's Beta with the market and as such a risk measure (Damodaran, 1999) meaning that a negative relationship with the valuation is expected. Finally, the positive sign of the 14th Component, which is primarily expressed by Altman's Z-Score, is also expected, as a decrease in this variable is correlated to a higher chance for the company to go bankrupt[60].

Components, which fundamentally are associated with the cash available to the company and the company's profitability, such as the first, the sixth, the ninth, the twelfth and the sixteenth one, bear a positive sign, as they tend to affect valuation in a highly significant (all of them except the ninth component are significant at the 0.01% level) and beneficial way. These results, besides being intuitive are also consistent with a plethora of studies that use them as measures for the valuation process (see for example the papers of Allayannis and Weston, 2001; Cooper and Priestley, 2016; Wilcox, 1984).

Similar positive effect is displayed by the components that are linked to higher corporate governance standards and better management practices (components eleven and thirteen), as well as those that link management performance with the way that managers are being rewarded (and whether the compensation reinforces an alignment of interests between shareholders and management (Component 17)), with the results being further reinforced by studies such as that of Brick et al. (2006), who find that compensation practices that are linked to incentive compensation (for example restricted stock instead of just salary), tend to increase the value created for investors. Similar are the results for the 15th component that also links management performance (Jensen's alpha) with higher quality in reporting, as it exemplified by the increased weight of the Big 4 auditors in the components.

[60] In 2007 Altman estimated the median of the companies near 1.8, which led him to believe that a financial crisis was imminent, and his estimations were proven correct two years later. Source: https://www.investopedia.com/terms/a/altman.asp

As expected, and in accordance with prior research (Fisher and Statman, 2003), the control variables for the US market with the mature companies (NASDAQ), GDP growth and Consumer Confidence Index have a positive impact upon the valuation. On the other hand, higher long-term interest rates, unemployment rate, inflation, output gap and political risk are associated with lower valuations. These findings are also consistent with the previous literature (see for instance Damodaran, 2015, and Bekaert et al. ,2016).

As previously noted, the US market can be described by fewer components. Alpha, as expressed in the 16th component, which is a measure of managerial performance (Cochrane, 2011), has a positive and highly significant effect on the valuation measure. Moreover, beneficial and vastly significant is the relationship between it and the various measures of profitability we have used in the original samples as those are expressed by the EBITDA (1st component), Tobin's Q (9th component) and Free Cash Flows (11th component). Similar is the effect of the Assets and the Revenues, as those are highlighted by the fifth and sixth component respectively.

At the other extreme (with a negative impact and highly significant), we have variables such as the Financial Crisis (Comp. 2), ROC (Comp. 13), which as we have already previously explained is a measure of profitability, however it can signal that the company might be an acquisition target and therefore linked to higher risk and the 15th component, which is described by the Beta (sensitivity to systematic risk) and the Effective Tax Rate (Grinblatt and Liu, 2008), explain that higher tax rates on profits lead to lower valuation estimates). Similar is the effect of higher WACC and Cost of Equity (Comp. 4). The variance to the company's stock, as it is depicted by R-square (Comp. 7) affects negatively the valuation, an idea that can be found in several papers (for instance Campbell, 2007). Finally, we have to note here that we provide no analysis for the 8th component. This is done deliberately, as no variable contributes more than the rest in forming this component and as such there is no prevalent characteristic for it to be interpreted.

One of the main goals in this thesis was to compare the risk profiles of the investors in the UK and US market. That is the reason why we contrast the results and we perform the analysis of both in the same section. There are a few points that can be made by observing the results on the panel data of the AIM and

NASDAQ markets. Firstly, the financial crisis and the regulation that predated and followed it are present in the results from both markets. This is not surprising if we consider the effect the crisis had on both the cost of equity and the cost of debt (Asker et al., 2015), as well as how it shaped the legislation, especially in regards to corporate governance (Adamson, 2012) and the risk perception of investors.

Another issue that is brought to the forefront is that, regardless of the market, investors require a high Tobin's Q, which is a variable that has gained significant traction over the years (Allayannis and Weston, 2001; Callahan et al., 2003; Gozzi et al., 2008) as a measure of company value or a measure of potential growth. Profitability is also a key variable in both markets, with measures of it being highlighted in components from both the AIM and NASDAQ sets. WACC and Cost of Equity are also very important for estimating the discount rate (with them also being the key determinants of components that explain a large portion of the total variability in their respective samples) as it has been exemplified many times throughout the literature (Booth, 2007; Krüger et al., 2015). In a similar fashion very important factors are the assets and the return on them, with the results pointing towards the same direction as studies such as that of Fama and French (2007) and Ljungqvist and Richardson (2003).

However, this is where the similarities between the two markets end. The analysis, besides the similarities, sheds light on the differences that exist between a market for young, growth companies, such as the AIM and one that encompasses more mature companies, namely the NASDAQ. Investors in the AIM enterprises, seem to be more affected by the reduced disclosure framework in which these companies operate, as explained by Gerakos et al. (2013). For that reason, a reputable auditor seems to be affecting the valuation of the companies in this set positively. This was expected as it has already been noted previously by De Franco et al. (2011), that companies that operate under reduced disclosure (this paper's focus is specifically on private enterprises however the main notion holds for that case as well), prefer reputable auditors as a signal of good reporting quality.

Another difference between the AIM and the NASDAQ, is that investors in the former, seem to take the probability that the company might default into special consideration. The fact that Z-Score is amongst the

factors in the UK sample but not in the US one is proof of that. On the other hand, leverage is important in the mature market, while it is not represented in the growth one. The final point that can be made, is for the Beta and the Total Beta. The former can be found in both sets, however the latter is only found with the AIM companies. Damodaran (1999), originally created this measure to facilitate appraisers in using comparative listed company Betas and enabling the incorporation of unsystematic risk for private enterprises. This type of enterprise is however closer to that of the companies within the AIM market (for the reasons we have mentioned in the previous sections). As such it was expected that Total Beta would be prominently featured in this analysis.

5.4 Private Companies – UK and US

5.4.1 Introduction

As we saw in the introductory section of the thesis, private enterprises constitute the backbone of both the UK and the US economies, given the relatively small size of global stock markets this is a phenomenon prevalent in most countries around the world. Regardless of that fact however, they have been severely under researched. To provide a better understanding on the reasons behind this, we turn towards the literature, which indicates that determining the value of a private firms is affected by two different groups of factors (Brav, 2009; Damodaran, 2012; Hope et al., 2013). The first one is that private companies' information availability is limited, as they operate in a less transparent reporting framework. This results in basic input valuation measures not being available to appraisers (for example private enterprises have no readily available price and Beta). The second group of factors can be linked to the purpose that the valuation is conducted for, with the IPOs and M&As being prime examples of this, and how the purpose is translated in premiums for control or illiquidity discounts.

Regardless of whether a company is publicly traded or private, the main valuation principles do not change. Investing in a private enterprise, is still expected to generate cash flows, which will need to be discounted

at an appropriate rate. Calculating such a rate, however, is highly problematic in the private companies setting (Damodaran, 2012). Neither the cost of equity nor the cost of debt, as well as the ratio between them, is readily available. The calculation is further hindered by the fact that, investors in private businesses have more often than not, all their wealth tied to their investment in the business.

Taking all of the above into consideration, and to tackle this issue, we included in the original variables of this research, the Total Beta, which as we have seen is a measure of relative risk and a substitute for the public companies' Beta, proposed originally by Damodaran (1999b), and further backed, primarily, by professional practitioners such as Butler and Pinkerton (2008). This variable appeared to be a significant contributor in the public companies (AIM) that face similar restrictions as the private ones, as shown by the fixed-effects regression analysis conducted previously. To determine whether this measure can be used as a substitute of Beta for private firms, we will examine this variable's effect further, as we will be using the PCA components as independent variables for this part of the analysis.

Another major issue we will also tackle is how control over the firm affects the valuation. Brau and Fawcett, (2006), argue that private company owners prefer to keep them from going public mainly due to control concerns, regardless of the size of the company, while they tend to do the opposite when they want to ease the process of an M&A. Brav (2009) explains that control considerations in private enterprises can even affect their capital structure, since stockholders in private firms do not issue new equity as they do not want to diminish their control over their businesses. This leads to a higher cost of equity, and private companies relying primarily on debt as a source of financing (an idea further reinforced by the study of Vanstraelen and Schelleman, 2017). Similarly, Michaely and Roberts (2012) find that the unique ownership structure in private companies affects the dividend policies in these firms. Inspired by studies such as these, we include a firm control variable in the empirical analysis (a binary variable), unique to the private companies,

where we examine whether more than 51% percent of the company's stock is being sold[61], and hence whether the owners of private firm are relinquishing their control over it.

To counter the issue of data availability, and to best represent the private firms in the thesis, we follow the most commonly employed practice, which is that of comparable companies[62]. The comparable method has been extensively used throughout the literature, and is also the preferred methodology employed by analysts as well, because as Liu et al. (2002) explain, the overarching idea behind it is that value can be viewed as a function of growing payoffs and a diminishing function of the risk associated with the firm. Others, such as Beatty, Riffe, and Thompson (1999) justified the use of comparable companies through more practical approaches, namely by looking at court cases and how the prices determined through the tax litigation system are in accordance with those proposed by the comparables methodology. Even staunch critics of the comparable method (such as Penman, 2010, or Kim and Ritter, 1999), admit that this method despite its drawbacks[63], is considerably easier to implement than a DCF methodology, which requires several variables as input (we will see however that DCF is still superior in producing more robust valuation estimates, with a mix of the two methods being the highest yielding approach).

The next question that should be answered, after having determined how we will approximate the private firms in this study, is which measure of value should be used to accurately represent these companies' worth. Although we have discussed the choice of the P/E ratio as the valuation ratio in the Data and Methodology sections, we feel it is important to provide some further insight as to why we chose this measure of value to approach the private companies' worth. P/E has been the recipient of significant

[61] The firm-control variable is binary and the value of (1) is assigned to it, when more than 51% percent of the company's stock is transferred during the M&A, else it gets the value (0).

[62] We have discussed how the comparable companies' method is applied in the Literature Review (2.3 Methods of Valuation). In that section we explain that in order for the appraiser to use this method of valuation, they need to identify companies within the same business as the one that needs to be valued, determine which variables will be used as proxies, get an estimate on those, and apply an average on those estimates to get the value of the company itself. In general, it is a method that is based on the notion that companies within the same industry will share the same risk profiles, as they face the same difficulties.

[63] Penman (2010) characterizes the comparable method as "*cheap and easy*", in the sense that it assumes an underlying market efficiency, which indicates that analysts' estimates are accurately and efficiently priced because they are based on prices that are "fair" according to the market. In many cases however, he argues, this is not the case as comparable companies are often mispriced. Kim and Ritter (1999), find that using comparables methodology is not appropriate to value initial public offerings, since comparable companies are not perfect matches in the sense that cash flows are not proportional, and risks are not similar.

supporting evidence in the literature. For example, Alford (1992) examines the measure's precision, when valuing firms that are in a similar industry, as those will generally have identical risk profiles, growth and accounting principles. His results indicate that P/E is an excellent method of valuation when comparable firms are used with an increased number of SIC digits as a criterion.

Further research even focused on how the P/E compares to other valuation ratios that are often employed by both academics and analysts. Cheng and McNamara (2000) and Brau and Fawcett (2006), indicate that P/E is better than other measures of valuation multiples (both of them look at other measures such as the P/B and the P/S). Liu et al. (2002), also attest to this notion, by further explaining that earnings measures produce more accurate results, than those based on cash flows, book value and sales. Similar are the results provided by Kim and Ritter (1999), who propose however some adjustments to further enhance the predictability capabilities of the P/E (namely using the forward instead of historical earnings).

A final issue to address, if only briefly at this point, is how we approached the illiquidity discount that private firms face (Damodaran, 2012). Although the several issues associated with private business valuation, will be approached through the variables (components) determined through the PCA for the public comparable firms, we account for the illiquidity in an alternate way, in the sense that we will measure the illiquidity through the transaction differences in the M&As of private firms and comparable transactions for public ones. To do that we base the methodology on two highly cited studies, Kaplan and Ruback (1995) and Officer (2007), which will be explained in greater detail in the sample description element of the empirical analysis. In short, we match the private companies' sample from the BVD Zephyr database twice. Primarily we match the private company transactions with the companies from the AIM and NASDAQ samples, to capture proxy estimates for the independent variables. Secondarily, we compare these private company M&As to similar public companies, which have undergone an M&A. The reasoning behind doing this, instead of simply using the P/E ratio from the private transactions as the dependent variable, is because by doing this we will be certain that the analysis will capture this important characteristic of illiquidity,

which would be difficult to approach in another way, as there is no variable that could be included in the components that can do so.

In the next section we will explain how the sample was created and calculate the discount in the final value of the private company M&A deals (representing the illiquidity discount faced by private enterprises). We will also present both the theoretical background, as well as, an overview of the distribution of the firms that we use as comparable to those included in the public companies' part of the thesis. In the empirical results part, we will present the final results, from regressing the AIM and NASDAQ components on the private companies' comparable P/E ratios drawn from public company M&A transactions (thereby effectively adjusting for the illiquidity discount). Then provide a comparison between private and public companies (which are a separate dataset of companies to the public ones we use to compare with the private ones to determine the illiquidity discount of our private companies' sample), and how those differ (or are similar) in terms of the characteristics that impact value the most. Figure 11 below provides a short summary of the process that will be followed.

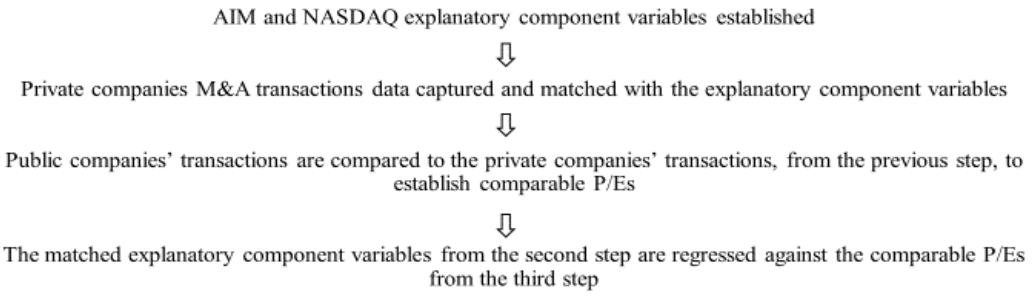


Figure 11: Summary table of the process followed, for the private companies

5.4.2 Sample and valuation measure

The sample consists of two separate streams, one focused on UK and another on US private companies, and spans the period 2004 to 2015, following the same chronological criteria used to construct the public companies' sample, in this thesis. For these years we acquired data on private companies' M&A deals from Zephyr[64], and more specifically by industry classification (UK SIC, US SIC), capturing the transaction value, the percentage of stock that has been transferred in the transactions, assets, revenues and number of employees for each company[65]. We supplemented these data from other sources, such as S&P Capital IQ and Bloomberg, from which we also collected M&A deal characteristics for public equivalent companies. The general criteria we used was to include private companies from all industry sectors with the target companies being from the US and the UK, with the exception of the financial, public, and regulated utilities sectors, as those are subject to specific regulations and other limitations that separate their valuation characteristics conceptually from other companies (Asker et al., 2015; Michaely and Roberts, 2012; Minnis, 2011). Applying the criteria yielded 8.184 companies for the UK, and another 5.227 for the US (Bureau van Dijk is primarily focused on EU companies' data, it does include however an adequate sample for other countries as well). On those we imposed further restrictions, as will be shown in the following paragraphs, in accordance with the related literature. Furthermore, we matched the remaining sample companies, with companies from the AIM and NASDAQ samples, which as a result of non-matching and criteria restrictions resulted in further reducing the final private companies' sample, which includes 2,192 for the UK and 2,503 for the US, as displayed in Table 26 that follows.

[64] Zephyr is a database specialized in private companies' transactions, published by Bureau Van Dijk.
[65] We also sampled several other variables, such as EBITDA, EV, Cash Flows, Dividends, etc., however for the purposes of this part of the thesis we focus on the ones mentioned above.

Public and Private Companies Sample Matching (Size)									
UK									
	Desc. Stat.			**Percentiles**					
	Mean	St. Dev.	Skewness	5%	25%	50%	75%	95%	
Assets	4.29	1.09	1.32	1.66	3.71	4.21	4.84	6.08	
No. of Companies				391	456	461	325	559	***2,192***
US									
	Desc. Stat.			**Percentiles**					
	Mean	St. Dev.	Skewness	5%	25%	50%	75%	95%	
Assets	3.44	2.04	1.47	0.36	2.08	3.29	4.61	7.09	
No. of Companies				128	541	324	402	1108	***2,503***

Table 26: Following the matching methodology proposed in previous studies, we matched the private companies we acquired from, primarily, Bureau Van Dijk's database with our companies from the original AIM and NASDAQ samples, so as to obtain the components that are assigned to those public companies. The matching has been done based on the companies' size, which is measured through the companies' assets, industry and year. The assets' percentiles (the natural logarithm) displayed in this table served to create "categories", by which the matching was done. The logarithmic value is displayed (following the example of Cooper and Priestley, 2016), as to provide a scaled view of the assets. In the lower row of each section of the table, the distribution of the companies throughout the various size percentiles can be seen. The industry distribution of the private companies can be found in Table 19 that follows.

For the UK portion of the sample we follow the studies of Brav (2009); Gerakos et al., (2013); Michaely and Roberts, (2012). As enacted in those studies, we exclude all other companies except the limited liability companies to ascertain that the Companies Act[66] applies to them, as this was the legislative act, we focused on for the AIM companies as well. Brav (2009) further explains that UK taxation laws do not differentiate on whether a company is listed or not, which further enhances the decision to compare the AIM and private firms in the UK, as they operate exactly under the same environment, both in terms of overall economy but also in terms of legislation.

In terms of company size, we filter the private companies to have a minimum of £0.7m in assets or having 50 or more employees, to be consistent with the AIM sample (as it is explained in the studies mentioned previously, this is the minimum assets listing requirement for the LSE of which AIM is part of). We also

[66] Both papers (Brav,2009; Michaely and Roberts, 2012) refer to several iterations of the Companies Act (1967, 1981). We use the 2006 version of the Companies Act, however the core principles for the companies remain the same, while the 2006 Act amended the previous ones by accounting for the changes imposed on financial reporting, in the wake of the Sarbanes-Oxley legislation in the US.

want sample companies to be able to be audited, as auditors are key elements for the public companies, and so we want to examine their effect on the private business sample. With that in mind, we exclude companies with less than £1m in sales[67]. This leaves us with 3.104 companies, which after matching with the AIM companies' sample, leaves us with our final 2,192 matched companies.

The US sample's construction is based upon several studies (Abudy et al., 2016; Asker et al., 2015; Cooper and Priestley, 2016; Gerakos et al., 2013; Hope et al., 2013; Minnis, 2011) that deal with different aspects of private companies. We exclude Canadian companies, companies from US territories outside the US (Guam) and companies with missing data. Furthermore, we impose size restrictions on the US companies selected, with the minimum set at $0.5m in assets, so as to have consistency with the NASDAQ companies' sample. This also allows us, in conjunction with the companies' industry and year that the M&A takes place, to match the private and public firms' transactions, when we finalize the samples based upon the illiquidity discount. If no match can be found, the observations are discarded. This process yielded 3.891 private companies, which have been matched to their counterparts from our original NASDAQ sample, and the end results yielded 2,503 companies. A sector distribution analysis for the firms that will remain in the end and serve as the final sample to be regressed with the AIM and NASDAQ derived PCA components, can be found in the Table 27 that follows.

[67] Not all companies above £1m are audited, that criterion according to the literature though increases the chances that they will be. The relevant legislation suggests that £6.5m is the required cut-off point for mandatory auditing. We should highlight also that AIM companies have Corporate Governance and LSE regulations to abide to, whereas that is not the case for private firms. We constructed our sample however in accordance to the related literature (Brav, 2009; Michaely and Roberts, 2012)

Private Companies' Sample Industry Distribution		
	UK	**US**
Industry (*UK SIC*) / (ICB)	*No. of Companies*	*No. of Companies*
Oil and Gas *(0000) / (0001)*	109	28
Basic Materials (*1000*)	110	139
Basic Industrials (*2000*)	393	266
Consumer Goods (*3000*)	206	343
Health Care (*4000*)	139	398
Consumer Services (*5000*)	988	626
Telecommunications (*6000*)	49	172
Utilities (*7000*)	66	89
Technology (*9000*)	132	442
Total No. of Companies	***2.192***	***2.503***

Table 27: Sample Firm Distribution of the matched companies, between AIM/NASDAQ and private firms from Bureau Van Dijk. The components that are tied to these companies, from our original PCA methodology, will be used as independent variables in the ensuing Multiple Linear Regression Analysis.

In order to construct the portfolios of private comparable companies we turned to similar research papers, for example that of (Fama and French, 1993), who document the effect variables, such as size[68], have on stock prices. The process begins by sorting the sample for the years 2004 – 2015 by size (measured through the assets or if there were a lack of assets data by the number of employees the company has. In our case, however, the assets for all companies were present). The median of the market the companies operate in is then used to categorize companies into different sized portfolio, based on the percentile they pertain to. The market for the private companies is determined based on their public equivalents and the type of the industry they are part of (determined by their SIC code).

As we have matched the private companies sample from BVD to the AIM and NASDAQ companies (and the components that stem from their characteristics), we are now ready to use these paired private companies as a base for the final element, we will need in order to conduct our MLR analysis, in order to

[68] We considered other measures of sorting suggested in the relevant literature, such as market-to-book, leverage, etc., however the consensus is that size and industry as matching criteria suffice for the matching process to be effective (Alford, 1992; Gerakos, Lang, and Maffett, 2013).

give prominence to the private companies' characteristics that shape the discount rate in their valuation, namely the dependent variable.

Dependent Variable (Illiquidity Discount Calculation)

Having defined how we created both the samples of private companies for the UK and the US, and how our original AIM and NASDAQ companies are matched to the private companies from BVD, we can finally move to the final part of the sample, that ties the rest of them together, that of the private companies' valuation, and explain how the matching between the private and public companies' M&A deals has been done, so as to form our dependent variable for the MLR analysis that follows. Before we discuss the sample itself, however, it is important to provide some theoretical background, as it is the intention to use the methodology suggested in the studies of Kaplan and Ruback (1995) and, primarily, Officer (2007), which will account for the illiquidity private companies' transactions are characterized by.

The paper of Kaplan and Ruback (1995) suggests that discounted cash flows provide a reliable estimate of market value in a highly leveraged transactions environment, which is the case with private companies as reflected in the related literature. To enable that conclusion, they compare transaction values to present value estimates of cash flows[69]. They find that DCF methods are equally as good as comparable methodologies, although the most accurate results in terms of valuation are produced through a combination of the two. What is most interesting, and most relevant to this thesis, however, is how they come to this conclusion. From the three distinct methods of comparable companies that are used, which are: 1) valuation multiple calculated from companies in the same industry as the firm being valued, 2) valuation multiple from companies involved in a similar transaction as the company valued and 3) valuation multiple from

[69] They use a technique called Compressed Adjusted Present Value (Compressed APV), values firms by discounting capital cash flows at the discount rate for an all equity firm.

companies in the same industry involved in a similar transaction to the company being valued, the second method produces the most accurate representations of value for these deals.

Building upon this idea, Officer (2007), proposes a variation of the second methodology proposed in the former paper, namely the comparable industry transaction method. To measure the illiquidity discount in private company M&A transactions, he creates portfolios of comparable acquisition transactions of public companies, with the criterion of selection for a public firm to be in the portfolio being, having the same two-digit SIC code as the unlisted target. Furthermore, he requires the public company deal values to fall within a 20% margin of the private company deal values. Acquisition prices are reported by either the seller or the buyer and are part of a publicly available record (data for these transactions can be found in public databases such as the one used in this thesis, namely Zephyr), however it is not easy to measure the discount premium, that constitutes the most important determinant of the fair value of the transaction. According to the author this impediment can be quelled through the comparison of acquisition multiples for unlisted and similar (in terms of industry or size) listed companies.

For the purpose of this thesis, and based upon these studies, the acquisition discount is calculated as the percent difference between the acquisition multiple (P/E) for the private company and the average corresponding multiple for the (size-determined) portfolio of the comparable public companies' acquisitions[70]. This effectively means that the discount can have both negative and positive values, with the former made possible if the multiple's level is lower than the portfolio average and the latter occurring when the P/E ratio is higher than the average. Officer (2007) explains that this calculation method allows the researcher to establish an estimation value for the multiple, that alleviates and adjusts for the problem of the transactions related to private companies being overestimated, due to the higher information

[70] Data for the public comparable M&A deals were found through Bloomberg and Capital IQ.

asymmetry associated with private companies' reduced disclosure environment, an idea that is prevalent throughout the literature (Asker et al., 2015; Gilje and Taillard, 2016)[71].

This idea will enable the isolation and determination of the real value discount, for illiquidity, in the transactions under examination. Similarly, to these previous studies we exclude the transactions where the multiple's value exceeds 100% of the difference between the public and private companies' transactions. The average discount for illiquidity for the private firms' sample, for both the UK and the US can be found in the table that follows (Table 28), the results are within the same range as that determined by prior literature (Brav, 2009; De Franco et al., 2011; Officer, 2007; Officer, Poulsen, and Stegemoller, 2009), namely broadly between 10% and 30%.

Valuation Multiple Difference Between Private and Public Companies		
Industry (*UK SIC*) / (ICB)	**UK**	**US**
Oil and Gas *(0000) / (0001)*	13%	9%
Basic Materials (*1000*)	19%	16%
Basic Industrials (*2000*)	17%	18%
Consumer Goods (*3000*)	22%	26%
Health Care (*4000*)	15%	15%
Consumer Services (*5000*)	24%	29%
Telecommunications (*6000*)	23%	23%
Utilities (*7000*)	27%	26%
Technology (*9000*)	34%	28%

Table 28: In this table the distribution of private companies' discount for illiquidity, over their respective sectors, is displayed, based on the calculation proposed by Officer (2007).

[71] Another idea proposed in Officer (2007) to combat the effects of information asymmetry is to examine the method of payment for the transaction, as it is explained that bidders will choose to pay in stock instead of cash. Unfortunately, we were limited on the amount of information we could amass regarding the method of payment for the transactions, and therefore this test was not included in the thesis.

In order to construct the portfolios of public comparable companies' M&As, we followed the same path as we did previously (focusing again on Fama and French, 1993), who document the effect variables, such as size[72], have on stock prices. Since this process has been described previously, we are also going to point out the main differences in what we used as matching criteria with the private companies that we matched with the AIM and NASDAQ ones. For this part, we sort the sample for the years 2004 – 2015 by assets, by value deal, by industry (determined by the SIC code) and by the time the M&A took place. This methodology is consistent with what is suggested primarily by Kaplan and Ruback (1995), as well as by Officer (2007). The descriptive statistics of the final matched M&A sample, that we use to determine the illiquidity discount can be found in the Table 29, that follows.

Public and Private Companies' M&A Matching (Dependent Variable)

UK								
	Desc. Stat.			**Percentiles**				
	Mean	St. Dev.	Skewness	5%	25%	50%	75%	95%
Assets	9.05	1.83	1.85	2.73	6.56	9.12	11.95	15.44
Deal Value	9.59	1.88	0.97	3.89	7.70	9.65	12.37	16.23
US								
	Desc. Stat.			**Percentiles**				
	Mean	St. Dev.	Skewness	5%	25%	50%	75%	95%
Assets	9.58	2.95	0.31	3.27	6.08	9.52	14.28	18.83
Deal Value	10.87	2.00	0.02	4.29	7.83	10.86	14.51	17.81

Table 29: This table displays the characteristics of the matched public and private companies' sample used to determine the illiquidity discount. The public companies' M&As were matched with the public ones, through size, value deal, industry and time of the M&A. Similarly, to what has been done previously we report the natural logarithm of the assets and the deal value of the matched sample.

[72] We considered other measures of sorting suggested in the relevant literature, such as market-to-book, leverage, etc., however the consensus is that size and industry as matching criteria suffice for the matching process to be effective (Alford, 1992; Gerakos, Lang, and Maffett, 2013).

After including the matching between the AIM and NASDAQ companies (through size, industry and year) with the transactions of the private companies, the final sample is comprised of 2.192 companies for the UK and another 2.503 for the US. For those we will use the, now adjusted for illiquidity as established previously by Officer (2007), the log value of the P/E ratio as a dependent variable, and the matched components determined previously by the PCA, as independent variables. We will further add the firm-control interests variable, to account for control issues, as well as the macroeconomic variables, as we want to capture the overall economic environment in which the companies operate in (both public and private). In the next section we present the empirical analysis results.

5.4.3 Results

As we have already explained in this part of the thesis, we will be regressing the valuation multiple (P/E) against a set of variables that is created to represent all the parts that are used by appraisers when they perform a valuation. In that sense, we have included a set of macroeconomic variables, as well as a measure for the control that is being transferred in the M&A deal. These variables complement the list of the components from the original Principal Component Analysis. By using the components, which are constructed of the original variables we chose based upon the existing literature on discount rates, we are bridging the gap between the private and public firms, as we approximate the former by using the matched equivalent public ones (as is the norm for the comparable method). The results from the regression analysis can be found in the Table 30 that follows.

Valuation Multiple (P/E Ratio)		
	UK	**US**
Variables		
Control Variables		
Consumer Confidence Index	-0.173	**0.230*** **

	(-0.23)	(-0.04)
GDP Growth	**0.295*** **	**0.266** **
	(-10.2)	(-0.10)
Short-term Int. Rates	-0.176	**-0.127*** **
	(-0.12)	(-0.034)
Long-term Int. Rates	-0.005	**-0.293*** **
	(-0.17)	(-0.11)
Yield Spread	-0.0191	0.014
	(-0.601)	(0.897)
ICRG Rating	**0.683*** **	0.015
	(-2.39)	(0.342)
Unemployment Rate	**-0.559***	-0.017
	(-30.35)	(-0.811)
Inflation	-0.00032	**-2.084*** **
	(-0.58)	(-0.25)
Output Gap	**-0.352** **	**-1.493*** **
	(-0.13)	(-0.20)
Company - Specific		
Controlling Interests	**0.270** **	**0.218*** **
	(-0.11)	(-0.03)
Component 1	**0.538*** **	**0.104*** **
	(-0.09)	(-0.01)
Component 2	0.027	**-0.150*** **
	(-0.03)	(0.03)
Component 3	-0.051	-0.644
	(-0.04)	(-1.24)
Component 4	-0.067	0.001
	(-0.07)	(-0.01)
Component 5	-0.013	0.003
	(-0.04)	(-0.01)
Component 6	-0.020	0.039
	(-0.04)	(-0.04)
Component 7	**-0.129** **	0.016
	(-0.05)	(-0.01)
Component 8	**0.118** **	0.028
	(-0.05)	(-0.01)
Component 9	-0.012	0.029
	(-0.07)	(-0.02)
Component 10	**-0.149** **	0.006
	(-0.05)	(-0.01)
Component 11	0.029	**0.050*** **

	(-0.01)	(-0.01)
Component 12	**0.150****	**-0.034****
	(-0.07)	(-0.01)
Component 13	0.014	**0.051****
	(-0.06)	(-0.02)
Component 14	**0.239*****	**0.060***
	(-0.08)	(-0.03)
Component 15	**0.122***	**-0.167*****
	(-0.06)	(-0.03)
Component 16	-0.094	**0.154*****
	(-0.09)	(-0.03)
Component 17	0.448	-
	(-0.49)	
Constant	-17.82	**-22.95*****
	(-27.89)	(-4.89)
Observations	**2192**	**2503**
R-squared	**0.183**	**0.159**

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 30: This table shows the results from the MLR of the macroeconomic, company-control as well as the components on the P/E multiple. Significance levels are denoted by the number of (*), with ***p<0.01, **p<0.05, *p<0.1 being significant at the 1%,5% and 10% respectively. Standard errors are in the parentheses. Constituents of each component are as detailed in Table 16.

We begin the analysis of the private companies' regressions with the set of control variables. Both the UK and the US private companies seem to be affected by the GDP growth, with 0.295 and 0.266 coefficient values respectively, although this effect seems to be stronger for the UK as the coefficient displays higher significance at the 1% level. Similarly, to the public companies, both their UK and US private counterparts' valuation are also affected in a negative manner by the output gap, something also consistent with the literature (Cooper and Priestley, 2009).

This is however where the similarities between the private enterprises from both countries stop, as the US private companies' valuation is significantly affected by more factors than their UK equivalents. US private firms seem to be positively affected by the CCI index at the highest degree, as the coefficient for it is highly

significant and positive. Similarly significant but negative is the interaction between the short (-0.127) and long term (-0.293) interest rates, which are a measure of liquidity towards the companies (Lekkos, 2003) and an indication of the prospects of the economy (Jiménez et al., 2014). Finally, US private companies seem also to be negatively affected by inflation (-2.084), with expectations for a higher level of it, being a well-established measure of appraisers' pessimism for the future performance of the overall economy (Durré and Giot, 2007).

UK private firms' valuations, on the other hand, seem to be more sensitive to changes in the political risk, as indicated by the positive and significant at the 1% level coefficient of the ICRG Rating (0.683), which indicates that appraisers in the UK value political stability more, than their US counterparts. The unemployment rate also appears to have an adverse effect on valuation, with the coefficient (-0.559) being negative and significant at the 10% level. The relationship between unemployment rate (among other macroeconomic variables) has been cited several times throughout the literature (Demirovic and Thomas, 2007; Gospodinov et al., 2014; Solow, 1980), which is expected as it is one of those factors that determines the future economic activity of a country.

Having discussed the macroeconomic variables, we move to another variable that was common between both the UK and the US samples, the Controlling Interests variable. As we have explained this parameter was inspired by the private companies literature (Asker et al., 2015; Brav, 2009; Hope et al., 2013; Michaely and Roberts, 2012), who explained that the control over the company is so important for their owners/managers, that it acts as a deterrent for private companies to go public. To examine this notion, we created a variable that examines whether more than half of the company's shares (51%) are transferred on completion of the M&A transaction. Consistent with prior research, we also find that the transfer of control, is valued as a positively significant factor in both the UK and the US companies, with the coefficients being 0.270 and 0.218 respectively. The only difference in the two is that for the US, control over the company is significant at the 1% level, while this drops to 5% for the UK companies.

We now move to the components, which are composed of the original variables matched from the public equivalent companies. For the UK, the analysis reveals that seven, out of the original seventeen components that were highlighted by the PCA, are significant in determining the value of a private firm comparable (2 at the 1%, 4 at the 5% and 1 at the 10% level). This number is slightly higher for the US regression that highlighted eight components out of the sixteen there were in total (5 at the 1%, 2 at the 5% and 1 at the 10%). These differences are reflective of the differences between the two economies, and will allow a determination of how investors in these countries, which share many similarities (Gerakos et al., 2013; Michaely and Roberts, 2012) in terms of their financial systems and corporate governance characteristics, view private entities.

The first component in both the UK and the US, based on the analysis conducted in the components section, reflects the ability of the company to generate cash flows, as the first component in the UK is characterized by the FCF and its US counterpart by the EBITDA. The coefficients for both components are positive and highly significant (0.538 for the UK and 0.104 for the US). As we have seen the cash flows that are available to be directed to either the firm or the investors are of particular importance to the investors. For instance, Weir, Laing, and Wright (2005), indicate that private companies with high free cash flows are usually targets of hostile takeovers. For the US especially, this characteristic, in the form of FCF to firm and FCF to equity, is further highlighted in the eleventh component, which has a highly significant positive coefficient (0.050).

Debt-to-equity is another characteristic that is common for both private companies' regressions. In particular, it appears as the seventh component for the UK (and also cost of debt appears in the eighth component), with a significant at the 5% level, negative coefficient of -0.129. Similarly, significant (at 5%) and negative is the coefficient for the US (-0.034). It is not surprising however, as it is a prominent feature of several studies dedicated to private companies. A prime example of this is the study of Brav (2009), who uses this ratio as a means of deducing that private firms rely primarily on leverage, as their managers / owners do not want to issue equity, as they are afraid of diminishing their control over the firm. As we have

already discussed, this is further enhanced by the fact that cost of equity is much more expensive for private firms and thusly less attractive for them. This notion is further enhanced by the fact that in neither of the regressions, is cost of equity found to be significant.

What is really found to be significant on the other hand, only for the UK private enterprises however, and has a positive impact on the valuation are the Assets of the company, which appears in both the eighth (0.118) (which also links the assets with the cost of debt) and the twelfth component (0.150). As we have seen assets are a proxy for size, and they are particularly important for any business, regardless of whether it is public or private. Especially for private firms, assets can act as a measure for mitigating the risk of investing in a company where accounting information is scarce (Brav, 2009; Cumming, Fleming, and Suchard, 2005).

Similarly, to evaluate the risk of investing in a firm Altman's Z-Score has an impact as described in the fourteenth component. According to the literature it is one of the most commonly employed methods to assess the viability of a private firm (Bajaj et al. 2001; Brav, 2009; Erhemjamts and Raman, 2012; Graham, Li, and Qiu, 2008). In this case it is found to be positive (0.239), as in the higher the score the less the probability of the company to default in the short term, and significant at the 1% level.

For the UK, we have two more components that are shown to significantly affect the valuation multiple. The first one is the measure of relative risk of a private firm proposed by (Damodaran, 1999a), namely the Total Beta, which is represented by the tenth component, that has a negative, significant impact at the firm's value (-0.149). This finding is very important, as this is one of the first academic studies that provides empirical evidence of this measure's ability to act as a replacement for the normal Beta, as we have explained in the beginning of the private companies section, despite the support it has found in the professional world in the USA (Butler and Pinkerton, 2008).

Finally, the fifteenth component is also revealed to affect valuation positively (0.122), which is primarily characterized by the existence of an auditor that reviews the firm's financial statements (a major auditing

firm for our public companies, however not necessarily so for the private ones). This finding is complementing previous research (Brav, 2009; Hope et al., 2013), by adding to the notion that auditing, although not particularly customary for private enterprises, increases the company's value, as it signals a higher quality of reporting, and a better control over the management's actions (this is the reason why the fifteenth component includes Jensen's Alpha, which represents the quality of management for a company).

Turning to the US portion of the regressions, the Financial Crisis, which is the single constituent of component 2, appears to have an impact on the valuation of private firms. This variable was included, as we wanted to deduce the effect that external shocks have on the value of firms, not only directly by affecting their key performance indicators (Calvet and Fisher, 2007; Perez-Gonzalez and Yun, 2013), but also indirectly, through the perception of investors. It seems that its effects are a pervasive phenomenon, as this variable appears to affect negatively and not only the US public firms but also the private ones, with its coefficient being highly significant (-0.150).

Both the thirteenth, dominated by ROC, and the sixteenth component, primarily affected by the Marginal-Profit-to-Capital, are measures of how well a company services its debt and how it utilizes its assets. Specifically for the latter, Cooper and Priestley (2016), who suggested the variable originally, argue that is one of the defining factors for the cost of capital in private firms, and subsequently their valuations. These results further support this notion as the MPK's coefficient is highly significant and positive (0.154). Similar are the results for the ROC, with a positive coefficient of 0.051, significant at 5%. ROC's importance for the US private firms is also a major finding of Asker et al. (2015).

The next component that exhibits an important impact on the valuation multiple is illustrated in the fourteenth component (0.060) and is the Insiders Stock Percentage. We have included this variable as a means to examine how linking firm and management performance affects the appraiser's view of the company. It appears that investors, want to make sure that managers will not follow their own goals, but they will be focusing on the company's growth, and as such this variable is significant at the 10% level.

This characteristic is prevalent in private firms as the owner is usually the manager of the firm and in that sense they will always strive to sustain growth for their company (Brav, 2009; Michaely and Roberts, 2012).

The final component that appeared in the US private companies' regression analysis is the fifteenth. With a negative and highly significant coefficient (-0.167), this component focuses primarily on the company's Beta and the effective tax rate. Both of those factors have been found throughout the literature to negatively affect the valuation of firms (Hong and Sarkar, 2007), and as they were also prominently featured in the US public companies, we expected them to appear for the private companies as well. Having presented the regression results for the private companies in the UK and US, we will now conclude the empirical analysis section by discussing how these results contrast to those from the public firms and attempt to create the profile of investors for the UK and the US.

5.4.4 Public and Private Firms' Comparison

For the last part of the analysis we will compare how different factors affect the valuation of public and private firms, in an attempt to find similarities and differences between the two. To assist us in this endeavor, we consolidated the results of the regression analysis for both public and private enterprises in the UK and US, and the results can be found in table 31, where we focus primarily on the effect of the macroeconomic variables on the companies' value, and table 32, where we compare how the components performed in the regressions.

Comparison of Public and Private Firms			
Panel A			
UK		**US**	
Public	**Private**	**Public**	**Private**
Macroeconomic Variables		**Macroeconomic Variables**	
Short-term Int. R.	GDP Growth	**CCI**	**CCI**
Yield Spread	ICRG Rating	**GDP Gr.**	**GDP Gr.**
Output Gap	Unempl. R.	**Long-Term Int.**	**Long-Term Int.**
	Output Gap	Yield Spread	Short-Term Int.
		ICRG Rating	**Inflation**
		Inflation	**Output Gap**

	Unempl. R.
	Output Gap

Table 31: In this table we compare the effect the macroeconomic variables have on public and private firms in the UK and the US. All similarities are highlighted in bold.

As can be seen, both private and public firms in the UK and the US, are affected by the possibilities of the economy to perform at a higher capacity and take advantage of all of its resources in order to increase production and subsequently the GDP. As we know from the literature (for example Cooper and Priestley, 2009, and Rangvid, 2006), the Output Gap has been one of the best macroeconomic variables to explain stock returns not only in the UK and the US, but also in other G7 countries. Similarly, GDP growth, Inflation and Long-Term Interest rates, have been extensively used in the literature as indicators of a country's performance (Chordia and Shivakumar, 2006; Duffee, 2006), in this thesis however they only seem to be important for the US sample of companies, both public and private. Interestingly, so did the Consumer Confidence Index (CCI), which as has been shown in studies such as that of Papapostolou, Nomikos, Pouliasis, and Kyriakou (2013), to act as an excellent proxy for investor sentiment.

There are, however, several differences in how macroeconomic variables affect the valuation. Beginning with the UK, public firms are affected by Short-Term Interest Rates and Yield spread, both of which showed little significance for the private entities, who seem to be more sensitive to changes in the GDP, the political stability of the country (ICRG Rating) and finally the Unemployment Rate. The yield spreads and short-term interest rates' findings are consistent with the literature. Vanstraelen and Schelleman (2017), report that so far there has been no empirical evidence to link these two variables to private firms' performance. There has been however, overwhelming support for this notion for public firms (see for example the paper of Chen and Zhao, 2009). GDP, ICRG and Unemployment affect all companies universally, even if we fail to find evidence of that for out UK public portion of the sample, as they point to potential expansion or contraction for the business cycle of the firms. In the US the situation is reversed, with these variables affecting the public but not the private firms.

As we proceed to contrast the industry and company characteristics of the UK and US samples, one cannot help but notice the difference in the number of components for private and public firms in both samples. Specifically, the UK sample is represented by 17 components for its public companies' sample, with 12 of them impacting valuation significantly, while it has only 7 components describing its private firms' value proxy. Similarly, for the US, public companies can be approached by 12 components, as opposed to the 8 of the private ones. As this thesis is unique in the way that the analysis is performed, and the valuation process is approached, the differences in the numbers of components cannot be explained through the literature. We can, however, hypothesize that as investors in public firms have more access to information regarding the companies they want to value, they increase subsequently the number of factors they take into consideration, with the results evident in the table 24 that follows. We need to point out at this point, that as part of this analysis we will look for the characteristics that private and public firms have in common, as the individual and unique to each set components have been extensively discussed in the previous sections of the empirical analysis.

Comparison of Public and Private Firms			
Panel B			
UK		**US**	
Public	**Private**	**Public**	**Private**
Components		**Components**	
Comp.1	**Comp.1**	**Comp.1**	**Comp.1**
Comp.2	Comp.7	**Comp.2**	**Comp.2**
Comp.3	Comp.8	Comp.4	**Comp.11**
Comp.6	**Comp.10**	Comp.5	Comp.12
Comp.9	**Comp.12**	Comp.6	**Comp.13**
Comp.10	**Comp.14**	Comp.7	Comp.14
Comp.11	**Comp.15**	Comp.8	**Comp.15**
Comp.12		Comp.9	**Comp.16**
Comp.13		**Comp.11**	
Comp.14		**Comp.13**	
Comp.15		**Comp.15**	
Comp.16		**Comp.16**	
Comp.17			

Table 32: In this table we present the cumulative results for both the UK and the US private and public companies on the components that affect their valuations. All similarities are highlighted in bold. The constituent of each component can be found in Table 16.

The first characteristic that is noteworthy, is how prevalent the cash flows are towards the firm or the investors, as exemplified by the significance of Comp.1 in both the UK and the US (for the US, it also appears in the 11th component, which is common between public and private firms). This component in both occasions, deals with the amount of money available to the firm, regardless of whether it is for reinvestment purposes or for payment towards the investors, as in the UK it represents the FCF and in the US the EBITDA, which as we have discussed previously is a proxy for the cash flows. This variable's importance is also evident throughout previous research (Maio, 2013; Rajan and Wulf, 2006; Sawicki and Shrestha, 2008), with the main idea behind it being that higher cash flows lead to either higher growth for the firm or to higher gains for the investor.

Specifically, for the UK companies, another similarity between public and private, is the importance of Total Beta. We included this variable in the original dataset, as we wanted to test the theory proposed by Damodaran (1999b), who suggested it as a good measure of the relative volatility of a company towards the market it operates in. Damodaran argued that it can be applied instead of the CAPM's Beta (which is also featured in the 10th component along with the Total Beta), in the cases where the Beta could not be calculated, mainly due to the absence of a readily available price for the company[73]. We find empirical evidence to support this notion, as the tenth component is significant for the valuation of both public and private firms.

Assets and the Z-Score, a measure of bankruptcy that also utilizes a firm's assets to determine the probability of it not being able to meet its obligations towards its creditors, are found also to be common between the two sample sets of companies in the UK. As we have pointed out previously both of those

[73] As we have explained we overcome the data unavailability issues connected to private firms, through the use of the components from the comparable AIM and NASDAQ firms.

variables have seen overwhelmingly supportive evidence (Brav, 2009; Connor and Korajczyk, 1993; Davis and Peles, 1993; Gerakos et al., 2013), regardless of whether a private or a public company is being examined. Both of them act as a significant indicator on whether an investment is risky or not. For the former, there is a valuation method the assets-based approach. Consequently, for this reason both of those characteristics are common for both public and private enterprises.

Finally, for the UK we have the last common component, for both categories of firms, the fifteenth component, which is mainly influenced by the auditors (in the private companies' case would be the existence of an auditor) and several management characteristics (Jensen's Alpha and how much stock insiders hold). We expected auditors to be important for the AIM, as the overwhelming majority of firms use them (Gerakos et al., 2013) suggest that almost all of the firms in his sample use one of the Big 4 auditing companies, something we also find in this study sample, in an attempt to signal a good reporting quality to potential investors. These findings are in accordance with the findings of De Franco et al. (2011), which suggest that using one of the Big 4 auditing companies would result in a higher valuation. Moreover, we provide further empirical evidence on the importance of good management signaling towards potential investors. Michaely and Roberts (2012) suggest that in the case of private firms, the fact that the managers are also the owners of the firm acts a signal that the interests of management and that of investors are aligned, which is a notion that this study further attests to.

Proceeding to the US sample, we can see the impact the Financial Crisis had on both private and public enterprises' valuation, as the second component is significant in both regressions. External shocks have been proven to impact the expectations of investors and subsequently they influence how careful or careless investors are (Perez-Gonzalez and Yun, 2013). The financial crisis of 2007 was an impactful event and as such we expected to see it in the determination of the companies' worth. What is interesting, however, is that although the aftermath of the crisis can be seen in the valuations of the US private firms the same cannot be said for the UK dataset, which together with the macroeconomic characteristics that are unique for them, leads us to believe that UK investors are less prone to "momentum" like effects (as in they do not

get affected by the global investor sentiment, but they are more focused on their investments), or that they might be more optimistic than their US counterparts. A final explanation to this phenomenon can also be the fact that AIM companies are generally younger and smaller than their US comparables.

The next similarity for the US private and public firms comes in the form of the thirteenth component, which is characterized by the Return-on-Capital variable. This ratio, points towards how profitable a company is, or as explained previously, how the earnings of the firm are utilized to service the debt of a company, as well as what the earnings potential is for investors. As it seems US investors, instead of using more traditional measures of the company's health, such as the assets of the Z-score which are employed by their UK equivalents, they focus on how well a company can perform as it appears by how well it can meet its obligations, and subsequently how fast they will be able to cash out their investment. This is an idea often encountered in the venture capital literature (see for example the study of Chahine, Filatotchev, and Wright, 2007), where venture capitalists (or other types of "angel" investors) attempt to take a private company public so as to be able to withdraw from it as soon as possible.

Finally, for the US sample, we have two components that have Beta as their interconnecting link but are also defined by other characteristics such as the tax rate and firm MPK and management performance (Jensen's Alpha). All of these variables are indicative of the investors' attempt to determine whether they can realize additional gains from their investment directly (MPK) but also indirectly, through potential tax gains. Moreover, these variables, combined together with the ones that define components 1,11 and 13, might be representative of how investors in the US, view investments in general, namely as having a higher risk potential and subsequently, in need of a higher level of monitoring.

As a concluding remark we can say that regardless of the country under examination, we find a great number of similarities between public and private firms. In the UK investors seem more concerned with the longevity of their investment, as indicated by the components that are similar to both public and private companies. In the US on the other hand they seem to be more concerned with management performance and the ability of their investment to service its long- and short-term obligations. These characteristics are

noteworthy as they reveal a pattern, namely that investors within a country examine public and private companies through the same prism, however this is altered between different countries, (regardless of their many similarities in this case).

6. Conclusions

6.1 Introduction

When we began working on the thesis, we had a specific set of goals in mind. Firstly, we wanted to examine the literature and consolidate the factors that have been shown to affect the discount rate in the valuation of private enterprises. To do that we amassed and combed through a significant number of studies, to determine which factors considered in each of them is important and what every one of them represents. We then distilled those, and extracted a set of variables, which were then used in the unique methodological approach (PCA) applied to determine how each of them contributed to the variability of the discount rate, as this was reflected in the valuation ratio, we chose to represent that rate.

The components formed by the PCA, are linear combinations of the original variables we used in the thesis. Their main characteristics are, that compared to the original variables they are reduced in number, making it easier for us to use, while simultaneously retaining most of the originals' variability (which in essence means that they retain most of the information associated with each original variable). We then reviewed the components and explained how each of them is formed, by focusing on those factors that attribute the most on their variation.

The components were then used as the independent variables, in a fixed effects regression analysis, to determine how and if they affect the value of a public firm, as this value was expressed through the use of the P/E valuation multiple. These same components were used as independent variables for the adjusted P/E multiple for a set of private firms, as the link between public and private firms is a well-established one

throughout the literature, and as public companies' characteristics are considered to be transferable to private ones. In the section that follows we present these results and contrast the public and private enterprises determining factors.

6.2 Contribution to Knowledge

The main purpose with this thesis, was to attempt to determine whether the plethora of factors that have been identified throughout the literature, can be reduced to such a number, which would most parsimoniously and accurately represent the value of a company, and thusly allow appraisers and academics to focus on those. In that sense, we wanted to create a framework of reference within which only the most important factors would be highlighted, and which could be used to determine the discount rate that should be considered and applied when investing in a private firm. To do that we used a unique methodology, as discussed previously, the results of which were then applied to a series of regressions initially for public and later for private businesses.

When applied to the UK portion of the sample dataset, PCA reduced the original 48 variable-sample, to 17 new ones. These components, as the new variables are called, were influenced by the FCF, the Financial Crisis and the legislation related to the companies in the sample, WACC and the Cost of Equity, ROE, Debt-to-Equity, EBITDA, Cost of Debt, Effective Tax Rate, Tobin's Q, R-Square, Beta and Total Beta. Furthermore, it highlighted other variables, Inventory-to-Sales and the Number of Insiders Holding Stock, Assets and ROA, Alpha, Insiders Stock Percentage, Z-Score, Earnings Yield and Compensation- to-Assets. (These variables serve as constituents of the components, an analysis of which can be found in Table 16.)

Similarly, for the US, the following variables formed the components: EBITDA, Capital Expenditure, Financial Crisis, Compensation-to-Assets, ROA, Cost of Equity and Cost of Debt, Assets, Net Operating Margin, R-Square, Tobin's Q, Risk Management Practices, Board Composition, FCF, Debt-to-Equity, IK, ROC, Insider's Stock Percentage, Effective Tax Rate, Beta, Alpha and MPK, (as shown in Table 16).

We then used the components from the public companies as independent variables, through the well-established link of public and private firms, in a set of regressions, where the dependent variable, was the valuation multiple from the M&As in the UK and US, over the same period as the initial samples adjusted for illiquidity. The results of these regressions are important, as we managed to express the risk associated with investing in a private UK or US firm with only a handful of variables, with the explanatory power for the model being 18.3% and 15.9% respectively. The main idea was to determine whether these characteristics of public firms are transferrable over to private businesses. In that sense, we managed to reduce the initial 48 variables to 8 for the UK private firms and 9 for their US equivalents.

Specifically, for the UK, we found that the FCF, the Debt-to-Equity, Assets, Cost of Debt, Total Beta, Z-Score, Auditors and Jensen's Alpha are important variables that need to be taken into consideration when determining the value of a private firm (or the constituent elements of the P/E multiple). For the US we have the EBITDA, Financial Crisis, FCF, Debt-to-Equity, Return on Capital, Percentage of Insiders Holding Stock, Beta, Tax Rate, MPK and Jensen's Alpha as well. As one can notice there is some overlap between the important variables in the UK and the US, however, they appear to be mostly relying on different factors.

The final goal of the thesis was to "build" the investor profiles in the UK and the US, and determine any potential differences between them, despite the fact that they operate in countries that are fairly similar, not only in terms of economic strength but also in other characteristics, such as the legal and governance systems. In the end, and with the unique methodology applied, we managed not only to highlight those variables that are important for the valuation of private firms in the UK and the US, but we also were able to find the differences between them. The results indicate that UK investors value more traditional variables (assets, Z-score, Auditors, Total Beta, Debt-to-Equity), to determine whether investing in a private company is worth it or not, while investors from the US are more preoccupied with whether the firm will be able to service its short-term debt, as indicated by the variables that are more heavily focused not only towards profitability, earnings and debt (EBITDA, FCF, MPK ROC, Debt-to-Equity). However, this can

also be attributed to the differential aspect of AIM and NASDAQ companies, as the former are younger and smaller than the latter.

In that sense, US investors appear to have a riskier profile, when it comes to investing in private enterprises than those from the UK. Both of them however, value whether or not they will be able to exercise control over their investment, and if their interests are aligned with those of the people managing the firms. In essence the variables we identified through the analysis, can be used to create the discount rate for investing in a private firm in the UK and the US.

6.3 Limitations of the thesis

The major limitations of the thesis are connected primarily to the same problem all researchers of private companies face. Although we endeavored to capture the majority of variables that were given prominence in the literature, there was a small number of them that did not get covered. This could be attributed to either data availability (for instance some papers suggested, for UK private data, the Financial Analysis Made Easy (FAME) database, or the SDC for the US (André, Khalil, and Magnan, 2007; Asker et al., 2015)), or most importantly to the qualitative nature of the characteristics highlighted in some of the studies, that were impossible to examine through the scope of this thesis. To exemplify this point, one has to think of papers, such as the one by Barclay, Holderness and Sheehan (2007), who examine whether the manager of a company follows an active or a passive strategy when making decisions, or the research of Saade (2015) who examines how investor sentiment on specific sectors affects the valuation of companies in them. As it is easily understandable, including such factors might further increase the accuracy of the methodology employed in the thesis, it would require however, a tremendous amount of resources, (database access, interviews, a team etc.) and a very extended time frame, for what would probably account for a very marginal improvement in the final results' accuracy.

6.4 Future Research

As we have set the basis on which variables better represent a company's true value, and how this should be reflected in the discount rate determined by appraisers through the valuation process, it would be interesting to test these results in different countries. Specifically, we believe that it would be beneficial to examine how these findings hold, within the European Union, where although countries have a general overlapping framework, both legislative but also in terms of reporting and governance, they also have many differences (for instance in taxation). This would further allow researchers to determine how investors from different countries respond to the various risk levels associated with investing in private firms.

Another interesting topic would be how the results from this study, are linked to others with more qualitative characteristics. For instance in the Literature Review, we mentioned how family ownership affects the value of a firm (Villalonga and Amit, 2006). The effect of variables such as these could be important and further enhance these results.

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Ethical Conduct of the Research – Certificate

Certificate of Ethics Review

Project Title:	Private Business Valuation: Defining an appropriate Discount Rate
User ID:	805682
Name:	Ioannis Tsalkamas
Application Date:	18/11/2016 12:38:48

You must download your certificate, print a copy and keep it as a record of this review.

It is your responsibility to adhere to the University Ethics Policy and any Department/School or professional guidelines in the conduct of your study including relevant guidelines regarding health and safety of researchers and University Health and Safety Policy.

It is also your responsibility to follow University guidance on Data Protection Policy:

- General guidance for all data protection issues
- University Data Protection Policy

You are reminded that as a University of Portsmouth Researcher you are bound by the UKRIO Code of Practice for Research; any breach of this code could lead to action being taken following the University's Procedure for the Investigation of Allegations of Misconduct in Research.

Any changes in the answers to the questions reflecting the design, management or conduct of the research over the course of the project must be notified to the Faculty Ethics Committee. **Any changes that affect the answers given in the questionnaire, not reported to the Faculty Ethics Committee, will invalidate this certificate.**

This ethical review should not be used to infer any comment on the academic merits or methodology of the project. If you have not already done so, you are advised to develop a clear protocol/proposal and ensure that it is independently reviewed by peers or others of appropriate standing. A favourable ethical opinion should not be perceived as permission to proceed with the research; there might be other matters of governance which require further consideration including the agreement of any organisation hosting the research.

GovernanceChecklist
A1-BriefDescriptionOfProject: Defining the discount rate for private business valuation, by examining private enterprise data.
A2-Faculty: PBS
A3-VoluntarilyReferToFEC: No
A5-AlreadyExternallyReviewed: No
B1-HumanParticipants: No
HumanParticipantsDefinition
B2-HumanParticipantsConfirmation: Yes

Certificate Code: 805B-6831-4BFC-DC36-B8B1-0491-9EA6-7589 Page 1

Ethical Conduct of the Research – Form UPR16

FORM UPR16

Research Ethics Review Checklist

Please include this completed form as an appendix to your thesis (see the Research Degree Operational Handbook for more information)

UNIVERSITY OF PORTSMOUTH

Postgraduate Research Student (PGRS) Information		**Student ID:**	up805682
PGRS Name:	Ioannis Tsalkamas		
Department:	Accounting and Financial Management	**First Supervisor:**	Richard Trafford
Start Date: (or progression date for Prof Doc students)	**01/02/2016**		

Study Mode and Route:					
	Part-time	☐	MPhil	☐	MD ☐
	Full-time	☒	PhD	☒	Professional Doctorate ☐

Title of Thesis:	Price to Earnings (P/E) Determinants and the Valuation of Private Firms: A Cross-Country Comparison
Thesis Word Count: (excluding ancillary data)	79602

If you are unsure about any of the following, please contact the local representative on your Faculty Ethics Committee for advice. Please note that it is your responsibility to follow the University's Ethics Policy and any relevant University, academic or professional guidelines in the conduct of your study

Although the Ethics Committee may have given your study a favourable opinion, the final responsibility for the ethical conduct of this work lies with the researcher(s).

UKRIO Finished Research Checklist:

(If you would like to know more about the checklist, please see your Faculty or Departmental Ethics Committee rep or see the online version of the full checklist at: http://www.ukrio.org/what-we-do/code-of-practice-for-research/)

a) Have all of your research and findings been reported accurately, honestly and within a reasonable time frame?	YES ☒ NO ☐
b) Have all contributions to knowledge been acknowledged?	YES ☒ NO ☐
c) Have you complied with all agreements relating to intellectual property, publication and authorship?	YES ☒ NO ☐
d) Has your research data been retained in a secure and accessible form and will it remain so for the required duration?	YES ☒ NO ☐
e) Does your research comply with all legal, ethical, and contractual requirements?	YES ☒ NO ☐

Candidate Statement:

I have considered the ethical dimensions of the above named research project, and have successfully obtained the necessary ethical approval(s)		
Ethical review number(s) from Faculty Ethics Committee (or from NRES/SCREC):		805B-6831-4BFC-DC36-B8B1-0491-9EA6-7589
If you have *not* submitted your work for ethical review, and/or you have answered 'No' to one or more of questions a) to e), please explain below why this is so:		
-		
Signed *(PGRS)*:	Ioannis Tsalkamas	**Date:** 24/08/2019